UNIT – II
THREE PHASE BALANCED AND UNBALANCED CIRCUITS

Topics: Three phase circuits: Phase sequence (A-B-C & A-C-B) of source and load- Star and delta connections-Relation between line and phase voltages and currents in balanced three phase source phasor diagrams-Analysis of three phase balanced load circuits- Calculation of Active, Reactive power, apparent power in balanced three phase systems.
Analysis of three phase unbalanced load circuits-Loop Method- Millman's Theorem method- Star Delta Transformation methods, phasor diagrams.

INTRODUCTION:

A system which utilizes only one winding and generates single alternating voltage and current is known as a single-phase system.

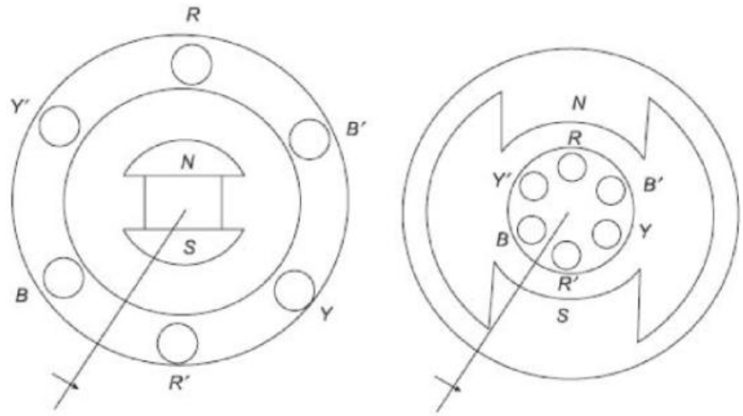
A system which utilizes more than one winding is called a poly-phase system. It produces as many induced voltages as the number of windings. The generation of electric power is, however, three-phase in practice because, even though it is possible to have any number of sources in a poly-phase system, the increase in the available power is not significant beyond the three-phase system.

ADVANTAGES OF THREE-PHASE SYSTEMS:

A three-phase system has a number of advantages over a single-phase system.
i) The output of a three-phase machine generating electricity is more than the output of a single-phase machine of the same size.
ii) The most commonly used three-phase induction motors are self starting. For single-phase motors, a separate starting winding is required.
iii) Electrical power transmission from the generating station to the places of use is done by transmission lines. It has been seen that three-phase power transmission is more economical than single-phase power transmission.
iv) The power factor of three-phase systems is better than that of the single-phase systems.
v) Single-phase supply can also be obtained from a three-phase supply.
vi) The instantaneous power in a single-phase system is fluctuating with time giving rise to noisy performance of single-phase motors. The power output of a symmetrical three-phase system is steady.
vii) For rectification of AC into DC, the DC output voltage becomes less fluctuating if the number of phases is increased.

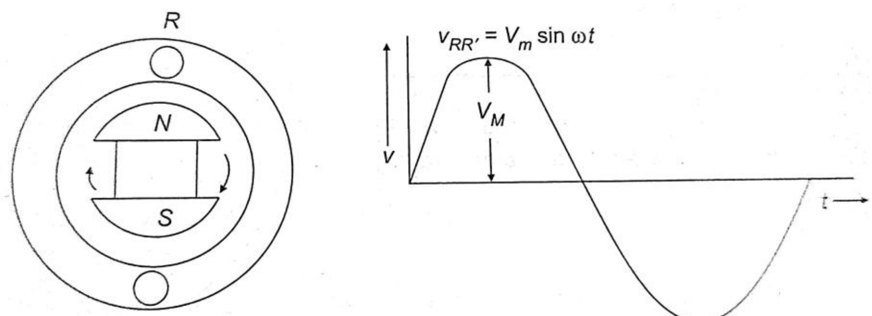
<u>GENERATION OF THREE PHASE VOLTAGES</u>

The Three-phase voltages can be generated in a stationary armature with a rotating field structure, or In a rotating armature with a stationary field as shown in Fig. (a) and (b).

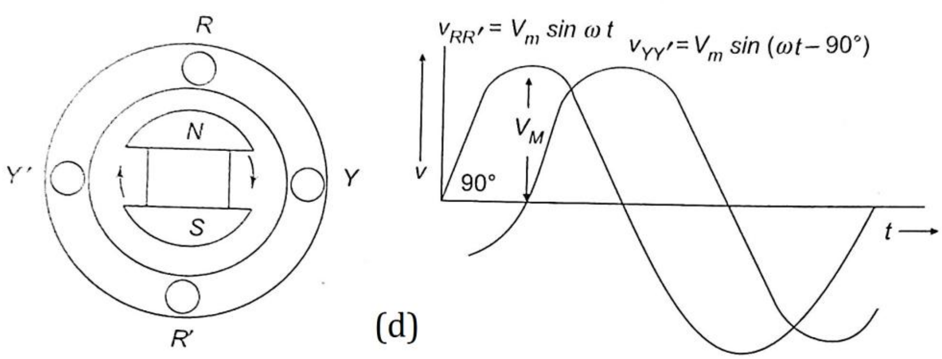


(a) Stationary Armature (b) Stationary Field

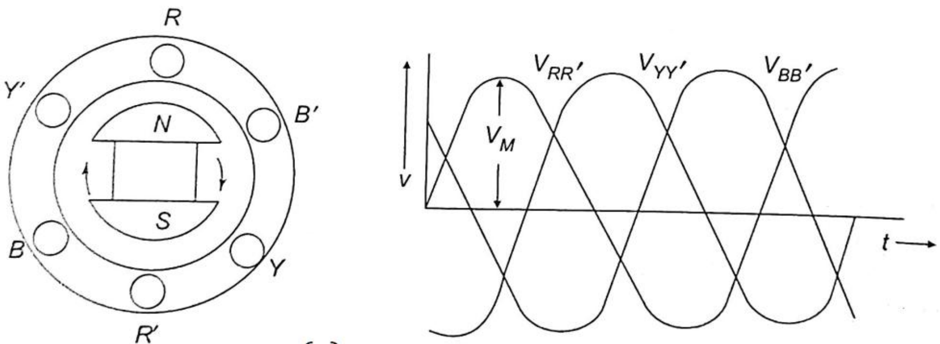
The Single-phase voltages and currents are generated by single-phase generators as shown in Fig.(c). The armature (here a stationary armature) of such a generator has only one winding, or one set of coils. In a two-phase generator, the armature has two distinct windings, or two sets of coils that are displaced 90° (electrical degrees) apart, so that the generated voltages in the two phases have 90° phase displacement as shown in Fig.(d). Similarly, three-phase voltages are generated in three separate but identical sets of windings or coils that are displaced by 120^0 electrical degrees in the armature, so that the voltages generated in them are 120^0 apart in time phase, This arrangement is shown in Fig.(e). Here RR' constitutes one coil (R-phase): YY' another coil (Y-phase), and BB' constitutes the third phase (B-phase). The field magnets are assumed in clockwise rotation.



(C)



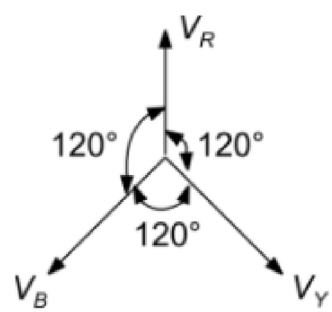
(d)



(e)

The voltages generated by a three-phase alternator is shown in Fig.(e). The three voltages are of the same magnitude and frequency, but are displaced from one another by 120^0. Assuming the voltages to be sinusoidal, we can write the equations for the instantaneous values of the voltages of the three phases. Counting the time from the instant when the voltage in phase R is zero.
The equations are

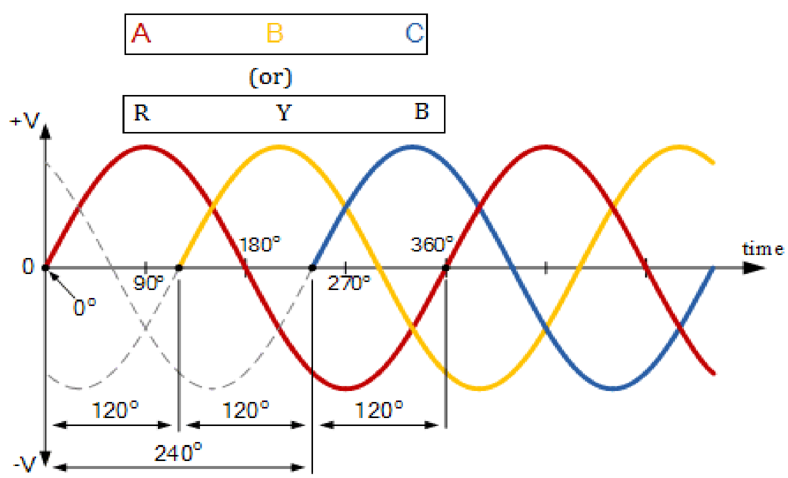
$$v_{RR}' = V_m \sin \omega t$$
$$v_{YY}' = V_m \sin (\omega t - 120°)$$
$$v_{BB}' = V_m \sin (\omega t - 240°)$$



PHASOR DIAGRAM

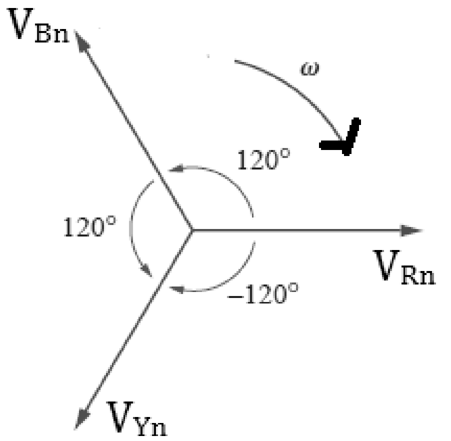
At any given instant, the algebraic sum of the three voltages must be zero..

NOTE: THREE PHASE VOLTAGE WAVEFORM

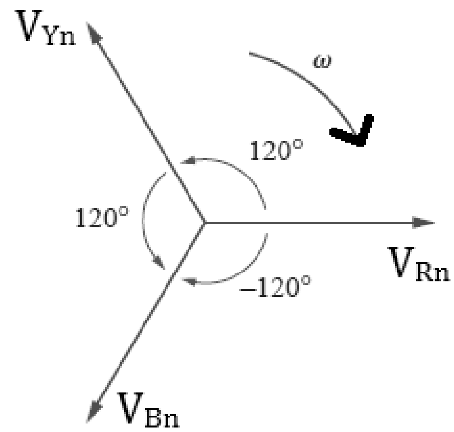


BASIC DEFINITIONS:

1. Phase Sequence: The sequence in which the voltages in the three phases reach the maximum positive value is called the phase sequence or phase order. From the phasor diagram of a three-phase system, it is clear that the voltage in the coil R attains maximum positive value first, next in the coil Y and then in the coil B. Hence, the phase sequence is R-Y-B.



(a) RYB or positive sequence



RBY or negative sequence

$v_{RR}' = V_m \sin \omega t$

$v_{YY}' = V_m \sin (\omega t - 120°)$

$v_{BB}' = V_m \sin (\omega t - 240°)$

$v_{RR}' = V_m \sin \omega t$

$v_{BB}' = V_m \sin (\omega t - 120°)$

$v_{YY}' = V_m \sin (\omega t - 240°)$

Example: Determine the phase sequence of the set of voltages

$$V_{an} = 200\cos\left(\omega t + 10^0\right) \qquad V_{bn} = 200\cos\left(\omega t - 230^0\right) \qquad V_{cn} = 200\cos\left(\omega t - 110^0\right)$$

Solution:

The voltages can be expressed in phasor form as

$$V_{an} = 200\angle 10^0;\ V_{bn} = 200\angle -230^0;\ V_{cn} = 200\angle -110^0$$

We notice that V_{an} leads V_{cn} by 120° and V_{cn} in turn leads V_{bn} by 120°.
Hence, we have an *acb (negative)* sequence.

2. Phase Voltage: The voltage induced in each winding is called the phase voltage.
3. Phase Current: The current flowing through each winding is called the phase current.
4. Line Voltage: The voltage available between any pair of terminals or lines is called the line voltage.
5. Line Current: The current flowing through each line is called the line current.
6. Symmetrical or Balanced System: A three-phase system is said to be balanced if the
(a) voltages in the three phases are equal in magnitude and differ in phase from one another by 120°, and
(b) currents in the three phases are equal in magnitude and differ in phase from one another by 120°.

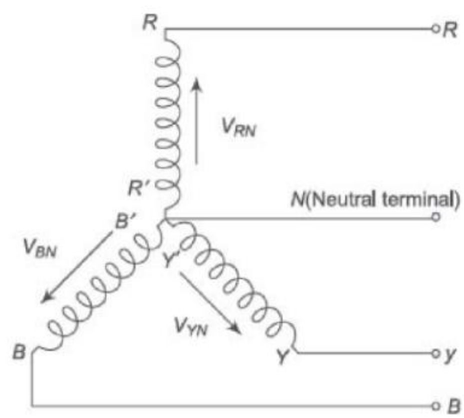
7. Balanced Load: The load is said to be balanced if loads connected across the three phases are identical, i.e., all the loads have the same magnitude and power factor.
8. Balanced supply: A set of three sinusoidal voltages (or currents) that are equal in magnitude but has a phase difference of 120° constitute a balanced three-phase voltage (or current) system.
9. Unbalanced supply: A three-phase system is said to be unbalanced when either of the three-phase voltages are unequal in magnitude or the phase angle between the three phases is not equal to 120°.
10. Unbalanced load: If the load impedances of the three phases are neither identical in magnitude nor in phase angle, then the load is said to be unbalanced.

STAR AND DELTA CONNECTIONS

In order to reduce the number of conductors, the three windings are connected in the following two ways: 1. Star or Wye connection 2. Delta or Mesh connection

STAR (OR) WYE CONNECTION:

In this connection, similar ends (start or finish) of the three phases are joined together within the alternator as shown in Fig. The common terminal so formed is referred to as the neutral point (N), or neutral terminal. Three lines are run from the other free ends (R, Y, B) to feed power to the three-phase load.



The Figure represents a three-phase, four-wire, star-connected system. The terminals R, Y, and B are called the line terminals of the source. The voltage between any line and the neutral point is called the phase voltage (V_{RN}, V_{YN}, and V_{BN}), while the voltage between any two lines is called the line voltage (V_{RY}, V_{YB}, and V_{BR}). The currents flowing through the phases are called the phase currents, while those flowing in the lines are called the line currents. If the neutral wire is not available for external connection, the system is called a three-phase, three-wire, star-connected system. The system so formed will supply equal line voltages displaced 120^0 from one another and acting simultaneously in the circuit like three independent single phase sources in the same frame of a three-phase alternator.

$$i_R = I_m \sin\theta$$

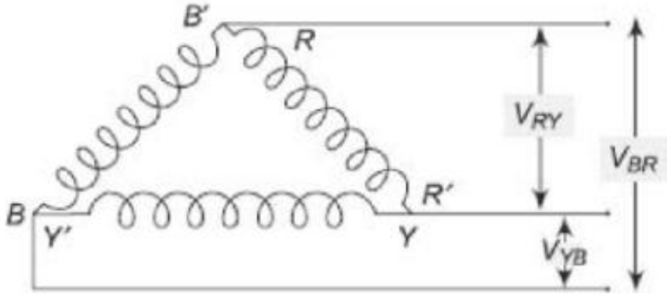
$$i_Y = I_m \sin(\theta - 120°)$$

$$i_B = I_m \sin(\theta - 240°)$$

$$i_R + i_Y + i_B = I_m \sin\theta + I_m \sin(\theta - 120°) + I_m \sin(\theta - 240°) = 0$$

DELTA OR MESH-CONNECTION:

In this method of connection, the dissimilar ends of the windings are joined together. i.e. R' is connected to Y, Y' to B and B' to R as shown in Fig.



The three line conductors are taken from the three junctions of the mesh or delta connection to feed the three-phase load. This constitutes a three-phase, three-wire, delta-connected system. Here there is no common terminal; only three line voltages V_{RY}, V_{YB}, and V_{BR} are available.

These line voltages are also referred to as phase voltages in the delta-connected system. When the sources are connected in delta, loads can be connected only across the three line terminals, R, Y and B. In general, a three-phase source, star or delta can be either balanced or unbalanced. A balanced three-phase source is one in which the three individual sources have equal magnitude, with 120^0 phase difference.

$$e_R = E_m \sin\theta$$

$$e_Y = E_m \sin(\theta - 120°)$$

$$e_B = E_m \sin(\theta - 240°)$$

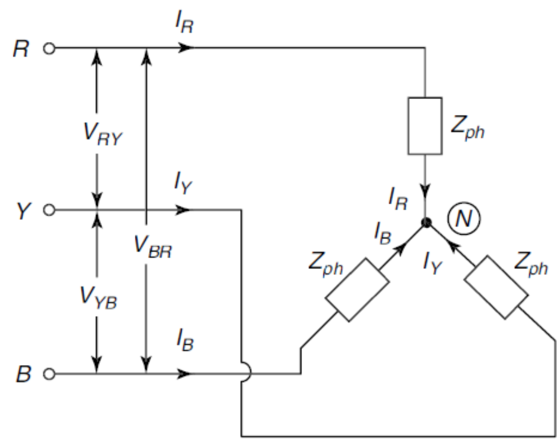
$$e_R + e_Y + e_B = E_m \sin\theta + E_m \sin(\theta - 120°) + E_m \sin(\theta - 240°) = 0$$

VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED STAR-CONNECTED SYSTEM

RELATION BETWEEN LINE VOLTAGE AND PHASE VOLTAGE:

Figure shows a balanced star-connected system

Since the system is balanced, the three-phase voltages V_{RN}, V_{YN}, and V_{BN} are equal in magnitude and differ in phase from one another by 120°.



Let $\quad V_{RN} = V_{YN} = V_{BN} = V_{ph}$

where V_{ph} indicates the rms value of phase voltage.

$$\mathbf{V}_{RN} = V_{ph}\angle 0°$$

$$\mathbf{V}_{YN} = V_{ph}\angle -120°$$

$$\mathbf{V}_{BN} = V_{ph}\angle -240°$$

Let $\quad V_{RY} = V_{YB} = V_{BR} = V_L$

where V_L indicates the rms value of line voltage.
Applying Kirchhoff's voltage law,

$$\begin{aligned}\mathbf{V}_{RY} &= \mathbf{V}_{RN} + \mathbf{V}_{NY}\\ &= \mathbf{V}_{RN} - \mathbf{V}_{YN}\\ &= V_{ph}\angle 0^\circ - V_{ph}\angle -120^\circ\\ &= (V_{ph} + j0) - (-0.5\,V_{ph} - j0.866\,V_{ph})\\ &= 1.5\,V_{ph} + j0.866\,V_{ph}\\ &= \sqrt{3}\,V_{ph}\angle 30^\circ\end{aligned}$$

Similarly, $$\mathbf{V}_{YB} = \mathbf{V}_{YN} + \mathbf{V}_{NB} = \sqrt{3}\,V_{ph}\angle 30^\circ$$

$$\mathbf{V}_{BR} = \mathbf{V}_{BN} + \mathbf{V}_{NR} = \sqrt{3}\,V_{ph}\angle 30^\circ$$

Thus, in a star-connected, three-phase system, $V_L = \sqrt{3}\,V_{ph}$ and line voltages lead respective phase voltages by 30°.

(i) Line voltage = $\sqrt{3}\,V_{Ph}$,

(ii) All line voltages are equal in magnitude and are displaced by 120°, and

(iii) All line voltages are 30° ahead of their respective phase voltages

RELATION BETWEEN LINE CURRENT AND PHASE CURRENT:

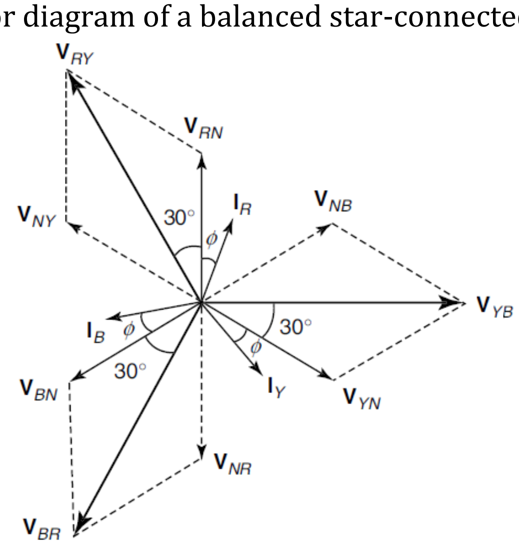
It is clear that line current is equal to the phase current.

$$I_L = I_{ph} = I_R = I_Y = I_B$$

It can be observed that the angle between the line (phase) current and the corresponding line voltage is $(30 + \phi)^\circ$ for a lagging load. Consequently, if the load is leading, then the angle between the line (phase) current and corresponding line voltage will be $(30 - \phi)^\circ$.

Phasor Diagram (Lagging Power Factor)

Figure shows the phasor diagram of a balanced star-connected inductive load.



Power:

The total power in a three-phase system is the sum of powers in the three phases. For a balanced load, the power consumed in each load phase is the same.

Total active power $P = 3 \times$ power in each phase $= 3\ V_{ph}\ I_{ph} \cos\phi$

In a star-connected, three-phase system,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi = \sqrt{3}\ V_L\ I_L \cos\phi$$

where ϕ is the phase difference between phase voltage and corresponding phase current.

Similarly, total reactive power $Q = 3V_{ph}\ I_{ph} \sin\phi = \sqrt{3}\ V_L I_L \sin\phi$

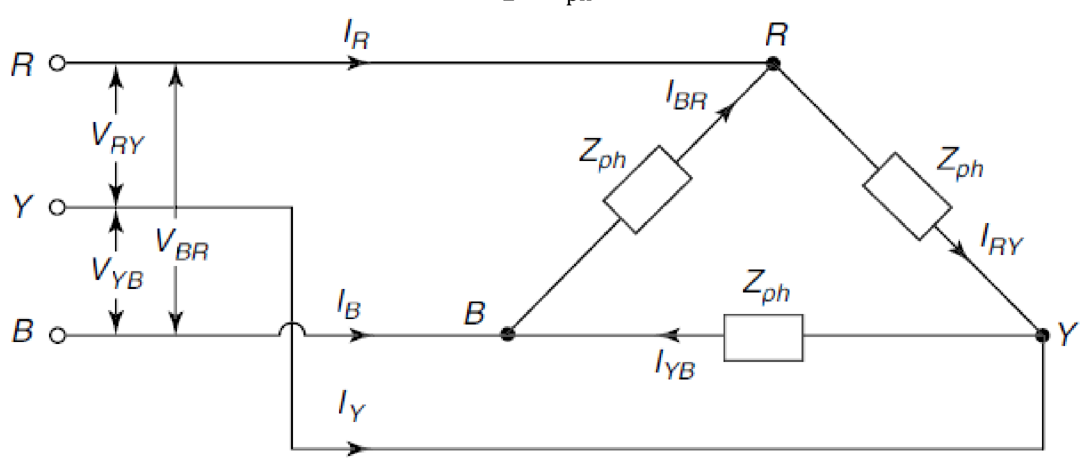
Total apparent power $S = 3V_{ph} I_{ph} = \sqrt{3} V_L I_L$

VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED DELTA-CONNECTED SYSTEM

RELATION BETWEEN LINE VOLTAGE AND PHASE VOLTAGE:

The Figure shows a balanced delta-connected system. From Fig., it is clear that only one phase is connected between any two lines. Hence, the voltage between any two lines (V_L) i.e. line voltage is equal to phase voltage (V_{ph}).

$$V_L = V_{ph}$$



Since the system is balanced, all phase voltages are equal but displayed by 120^0.

$$V_{RY} = V_{YB} = V_{BR} = V_L = V_{ph}$$

RELATION BETWEEN LINE CURRENT AND PHASE CURRENT:

Since the system is balanced, the three-phase currents I_{RY}, I_{YB}, and I_{BR} are equal in magnitude but differ in phase from one another by 120°.

Let $$I_{RY} = I_{YB} = I_{BR} = I_{ph}$$

where I_{ph} indicates rms value of the phase current.

$$\mathbf{I}_{RY} = I_{ph}\angle 0°$$
$$\mathbf{I}_{YB} = I_{ph}\angle -120°$$
$$\mathbf{I}_{BR} = I_{ph}\angle -240°$$

Let $$I_R = I_Y = I_B = I_L$$

where I_L indicates rms value of the line current.

Applying Kirchhoff's current law,

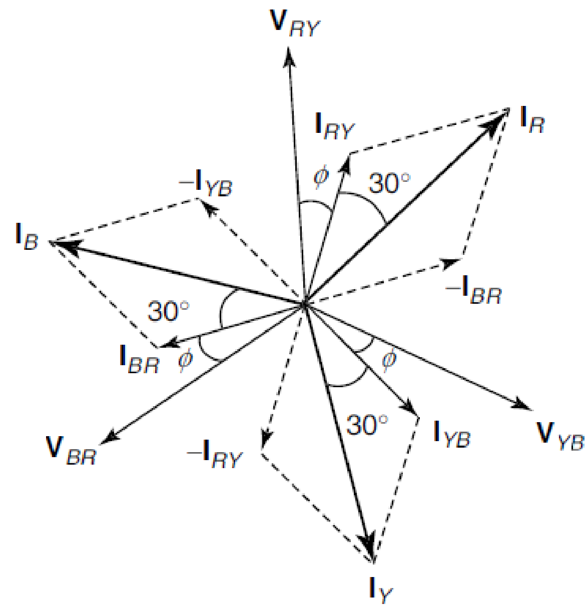
$$\mathbf{I}_R + \mathbf{I}_{BR} = \mathbf{I}_{RY}$$
$$\mathbf{I}_R = \mathbf{I}_{RY} - \mathbf{I}_{BR}$$
$$= I_{ph}\angle 0° - I_{ph}\angle -240°$$
$$= (I_{ph} + j0) - (-0.5 I_{ph} + j0.866 I_{ph})$$
$$= 1.5 I_{ph} - j0.866 I_{ph}$$
$$= \sqrt{3} I_{ph} \angle -30°$$

Similarly, $$\mathbf{I}_Y = \mathbf{I}_{YB} - \mathbf{I}_{RY} = \sqrt{3} I_{ph} \angle -30°$$
$$\mathbf{I}_B = \mathbf{I}_{BR} - \mathbf{I}_{YB} = \sqrt{3} I_{ph} \angle -30°$$

Thus, in a delta-connected, three-phase system, $I_L = \sqrt{3} I_{ph}$ and line currents lag behind the respective phase currents by 30°.

PHASOR DIAGRAM (LAGGING POWER FACTOR):

The Figure shows the phasor diagram of a balanced delta connected system.



Power:

$$P = 3V_{ph}I_{ph}\cos\phi$$

In a delta-connected, three-phase system,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3\times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

Total reactive power $Q = 3\, V_{ph} I_{ph} \sin\phi = \sqrt{3}\, V_L I_L \sin\phi$

Total apparent power $S = 3\, V_{ph} I_{ph} = \sqrt{3}\, V_L I_L$

BALANCED Y/Δ AND Δ/Y CONVERSIONS

Any balanced star-connected system can be converted into the equivalent delta-connected system and vice versa. For a balanced star-connected load,

$$\text{Line voltage} = V_L$$
$$\text{Line current} = I_L$$
$$\text{Impedance/phase} = Z_Y$$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\sqrt{3}\, I_L}$$

For an equivalent delta-connected system, the line voltages and currents must have the same values as in the star-connected system, i.e.,

$$\text{Line voltage} = V_L$$
$$\text{Line current} = I_L$$
$$\text{Impedance/phase} = Z_\Delta$$

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\frac{I_L}{\sqrt{3}}} = \sqrt{3}\frac{V_L}{I_L} = 3Z_Y$$

$$Z_Y = \frac{1}{3}Z_\Delta$$

Thus, when three equal phase impedances are connected in delta, the equivalent star impedance is one-third of the delta impedance.

COMPARISON BETWEEN STAR AND DELTA CONNECTIONS

Star Connection	Delta Connection
1. $V_L = \sqrt{3}\, V_{ph}$	1. $V_L = V_{ph}$
2. $I_L = I_{ph}$	2. $I_L = \sqrt{3}\, I_{ph}$
3. Line voltage leads the respective phase voltage by 30°.	3. Line current lags behind the respective phase current by 30°
4. Power in star connection is one-third of power in delta connection.	4. Power in delta connection is 3 times of the power in star connection.
5. Three-phase, three-wire and three-phase, four-wire systems are possible.	5. Only three-phase, three-wire system is possible.
6. The phasor sum of all the phase currents is zero.	6. The phasor sum of all the phase voltages is zero.

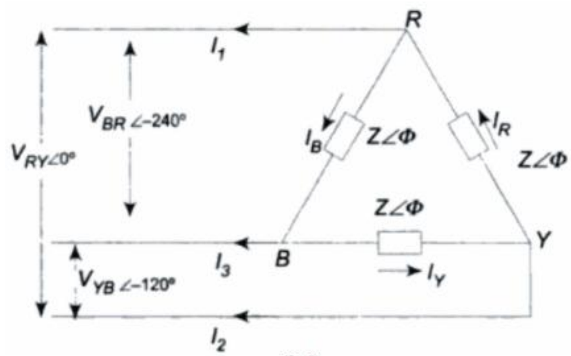
ANALYSIS OF THREE PHASE BALANCED LOAD CIRCUITS

a) Balanced Three-Phase system – Delta load:

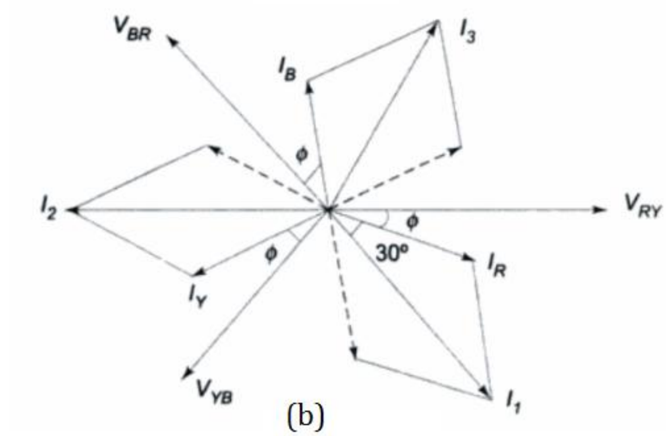
The Figure (a) shows a three-phase. three-wire, balanced system supplying power to a balanced three-phase delta load. The phase sequence is RYB.
Let us assume the line voltage $V_{RY} = V\angle 0^0$ as the reference phasor. Then the three source voltages are given by

$$V_{RY} = V\angle 0°\ \text{V}$$
$$V_{YB} = V\angle -120°\ \text{V}$$
$$V_{BR} = V\angle -240°\ \text{V}$$

These voltages are represented by phasors in Fig(b). Since the load is delta-connected, the line voltage of the source is equal to the phase voltage of the load. The current in phase RY, I_R will lag (lead) behind (ahead of) the phase voltage V_{RY} by an angle Φ as dictated by the nature of the load impedance. The angle of lag of I_Y with respect to V_{YB}, as well as the angle of lag of I_B with respect to V_{BR} will be Φ as the load is balanced. All these quantities are represented in Fig.(b).



(a)



(b)

If the load impedance is $Z\angle\phi$, the current flowing in the three load impedances are then

$$I_R = \frac{V_{RY}\angle 0}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$$

$$I_Y = \frac{V_{YB}\angle -120}{Z\angle\phi} = \frac{V}{Z}\angle -120^\circ - \phi$$

$$I_B = \frac{V_{BR}\angle -240}{Z\angle\phi} = \frac{V}{Z}\angle -240^\circ - \phi$$

The line currents are $\sqrt{3}$ times the phase currents, and are 30° behind their respective phase currents.
∴ Current in line 1 is given by

$$I_1 = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-\phi - 30^\circ), \text{ or } I_R - I_B \text{ (phasor difference)}$$

Similarly, the current in line 2

$$I_2 = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-120 - \phi - 30^\circ),$$

$$\text{or } I_Y - I_R \text{ (phasor diference)} = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-\phi - 150)^\circ$$

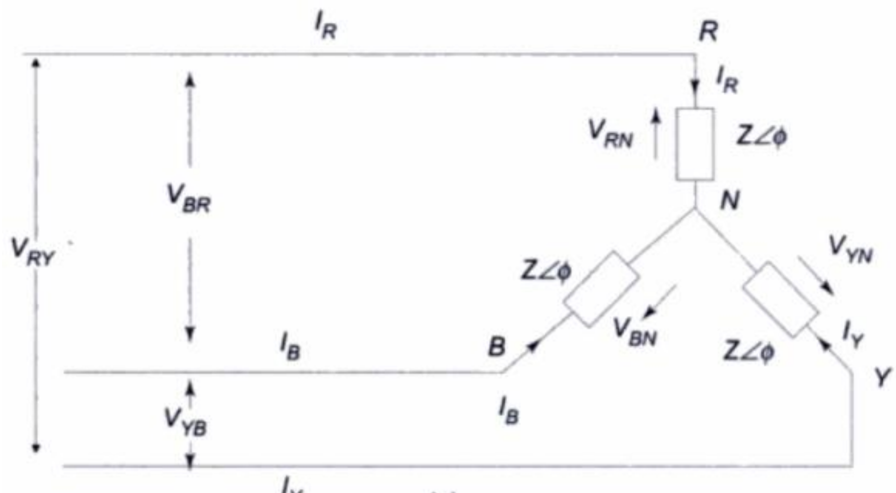
$$I_3 = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-240 - \phi - 30)^\circ, \text{ or } I_B - I_Y \text{ (phasor difference)}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle(-270 - \phi)$$

b) Balanced Three-Phase system – Star load:

The Figure(a) shows a three-phase, three-wire system supplying power to a balanced three phase star connected load. The phase sequence RYB is assumed.

In star connection, whatever current is flowing in the phase is also flowing in the line. The three line (phase) currents are I_R, I_Y, and I_B.



(a)

V_{RN}, V_{YN} and V_{BN} represent three phase voltages of the network, i.e. the voltage between any line and neutral. Let us assume the voltage $V_{RN} = V\angle 0°$ as the reference phasor. Consequently, the phase voltage

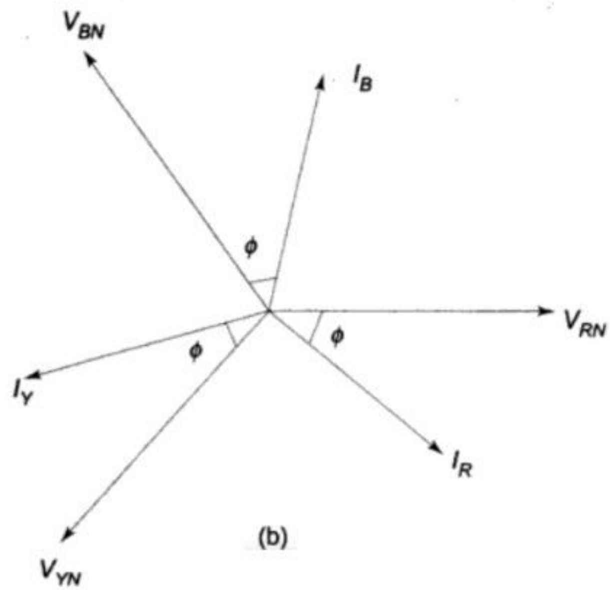
$$V_{RN} = V\angle 0°$$
$$V_{YN} = V\angle -120°$$
$$V_{BN} = V\angle -240°$$

$$I_R = \frac{V_{RN}}{Z\angle\phi} = \frac{V\angle 0}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle -\phi$$

$$I_Y = \frac{V_{YN}}{Z\angle\phi} = \frac{V\angle -120}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle -120° - \phi$$

$$I_B = \frac{V_{BN}}{Z\angle\phi} = \frac{V\angle -240}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle -240° - \phi$$

As seen from the above expressions, the currents, I_R, I_Y and I_B, are equal in magnitude and have a 120° phase difference. The disposition of these vectors is shown in Fig.:(b). Sometimes, a 4th wire, called neutral wire is run from the neutral point, if the source is also star-connected. This gives three-phase, four-wire star-connected system. However, if the three line currents are balanced, the current in the fourth wire is zero; removing this connecting wire between the source neutral and load neutral is, therefore, not going to make any change in the condition of the system. The availability of the neutral wire makes it possible to use all the three phase voltages, as well as the three line voltages. Usually, the neutral is grounded for safety and for the design of insulation.



(b)

It makes no difference to the current flowing in the load phases, as well as to the line currents, whether the sources have been connected in star or in delta, provided the voltage across each phase of the delta connected source is $\sqrt{3}$ times the voltage across each phase of the star-connected source.

PHASE AND LINE VOLTAGES/CURRENTS FOR BALANCED THREE-PHASE SYSTEMS

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \frac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

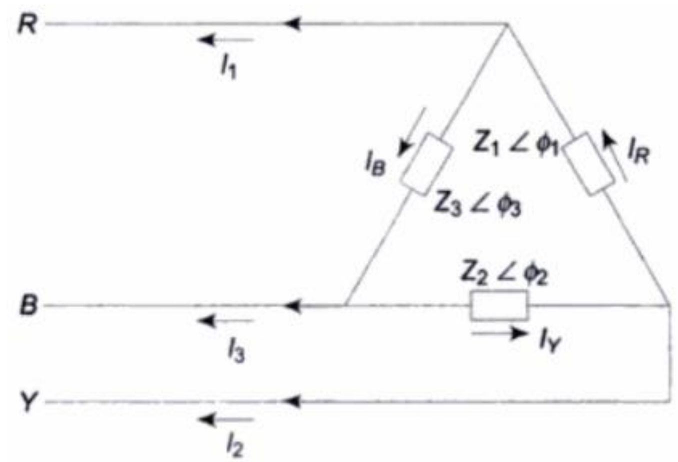
ANALYSIS OF THREE PHASE UNBALANCED LOAD CIRCUITS

An unbalance exists in a circuit when the impedances in one or more phases differ from the impedances of the other phases. In such a case, line or phase currents are different and are displaced from one another by unequal angles. It is enough to solve problems, considering one phase only on balanced loads; the conditions on other two phases being similar. Problems on unbalanced three-phase loads are difficult to handle because conditions in the three phases are different. However, the source voltages are assumed to be balanced. If the system is a three-wire system, the currents flowing towards the load in the three lines must add to zero at any given instant. If the system is a four-wire system, the sum of the three outgoing line currents is equal to the return current in the neutral wire. In practice, we may come across the following unbalanced loads:

(i) Unbalanced delta-connected load
(ii) Unbalanced three-wire star-connected load
(iii) Unbalanced four-wire star-connected load..

i) UNBALANCED DELTA-CONNECTED LOAD:

The Figure shows an unbalanced delta-load connected to a balanced three-phase supply.



The unbalanced delta-connected load supplied from a balanced three-phase supply does not present any new problems because the voltage across the load phase is fixed. It is independent of the nature of the load and is equal to the line voltage of the supply. The current in each load phase is equal to the line voltage divided by the impedance of that phase. The line current will be the phasor difference of the corresponding phase currents, taking V_{RY} as the reference phasor.

Assuming RYB phase sequence, we have

$$V_{RY} = V \angle 0° \text{ V},\ V_{YB} = V \angle -120° \text{ V},\ V_{BR} = V \angle -240° \text{ V}$$

Phase currents are

$$I_R = \frac{V_{RY}}{Z_1 \angle \phi} = \frac{V \angle 0}{Z_1 \angle \phi_1} = \left| \frac{V}{Z_1} \right| \angle -\phi_1 \quad \text{A}$$

$$I_Y = \frac{V_{YB}}{Z_2 \angle \phi_2} = \frac{V \angle -120}{Z_2 \angle \phi_2} = \left| \frac{V}{Z_2} \right| \angle -120^\circ - \phi_2 \quad \text{A}$$

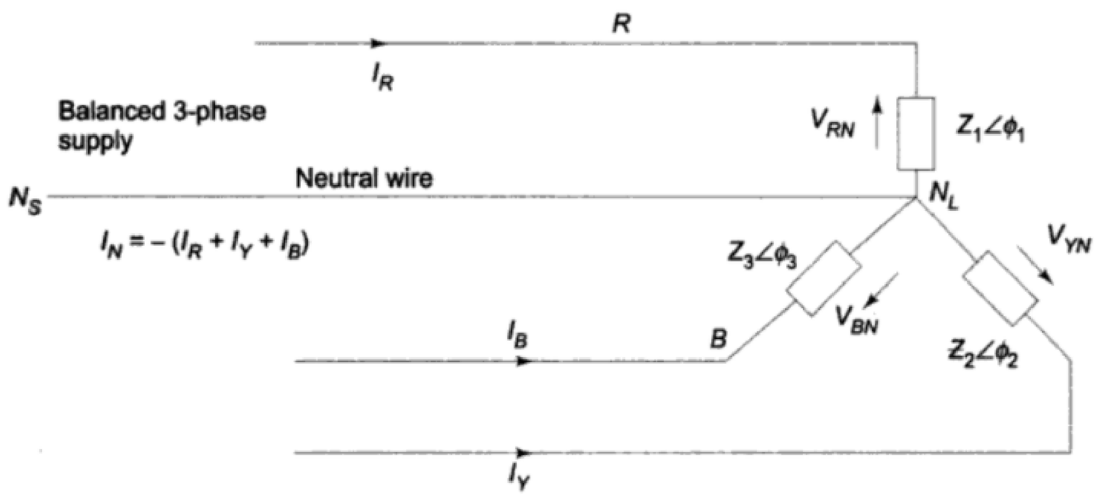
$$I_B = \frac{V_{BR}}{Z_3 \angle \phi_3} = \frac{V \angle -240}{Z_3 \angle \phi_3} = \frac{V}{Z_3} \angle -240^\circ - \phi_3 \quad \text{A}$$

The three line currents are

$$I_1 = I_R - I_B \text{ phasor difference}$$
$$I_2 = I_Y - I_R \text{ phasor difference}$$
$$I_3 = I_B - I_Y \text{ phasor difference}$$

ii) UNBALANCED FOUR WIRE STAR-CONNECTED LOAD:

The Figure shows an unbalanced star load connected to a balanced 3-phase, 4-wire supply.



The star point, N_L, of the load is connected to the star point, N_S of the supply. It is the simplest case of an unbalanced load because of the presence of the neutral wire; the star points of the supply Ns (generator) and the load N_L are at the same potential. It means that the voltage across each load impedance is equal to the phase voltage of the supply (generator), i.e. the voltages across the three load impedances are equalized even though load impedances are unequal, However, the current in each phase (or line) will be different. Obviously, the vector sum of the currents in the three lines is not zero, but is equal to neutral current. Phase currents can be calculated in similar way as that followed in an unbalanced delta-connected load.

Taking the phase voltage $V_{RN} = V\angle 0°$ V as reference, and assuming RYB phase sequences, we have the three phase voltages as follows:

$$V_{RN} = V \angle 0° \text{ V}, \; V_{YN} = V \angle -120° \text{ V}, \; V_{BN} = V \angle -240° \text{ V}$$

The phase currents are

$$I_R = \frac{V_{RN}}{Z_1} = \frac{V\angle 0^\circ}{Z_1\angle\phi_1}\ A = \left|\frac{V}{Z_1}\right|\angle -\phi_1\ \text{A}$$

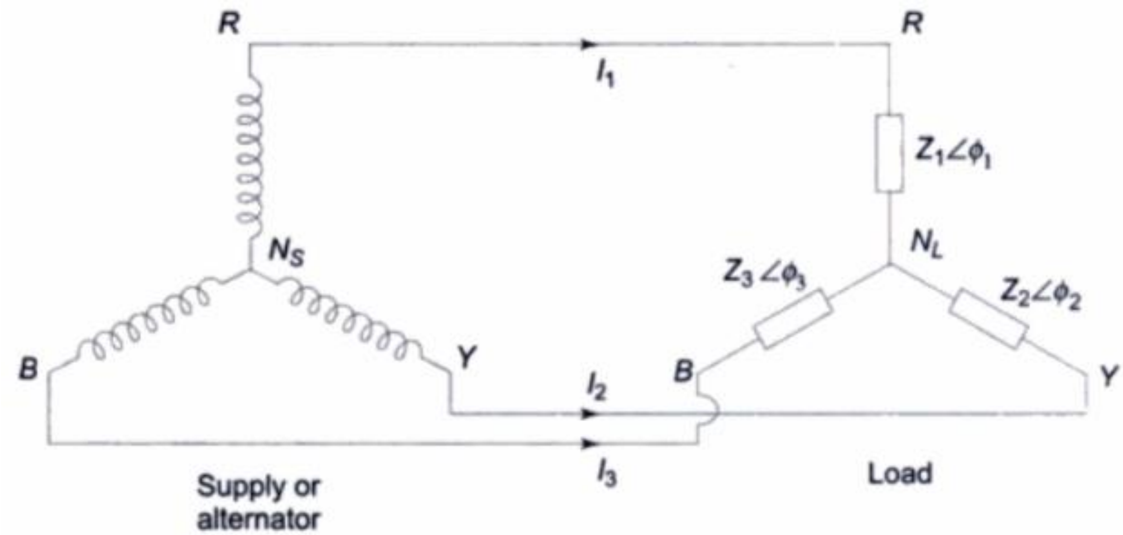
$$I_Y = \frac{V_{YN}}{Z_2} = \frac{V\angle -120}{Z_2\angle\phi_2}\ A = \left|\frac{V}{Z_2}\right|\angle -120^\circ - \phi_2\ \text{A}$$

$$I_B = \frac{V_{BN}}{Z_3} = \frac{V\angle -240}{Z_3\angle\phi_3}\ A = \left|\frac{V}{Z_3}\right|\angle -240^\circ - \phi_3\ \text{A}$$

Incidentally, I_R, I_Y and I_B are also the line currents; the current in the neutral wire is the vector sum of the three line currents.

iii) UNBALANCED THREE WIRE STAR-CONNECTED LOAD:

In a three-phase, four-wire system if the connection between supply neutral and load neutral is broken, it would result in an unbalanced three-wire star-load. This type of load is rarely found in practice, because all the three wire star loads are balanced. Such system is shown in Fig.

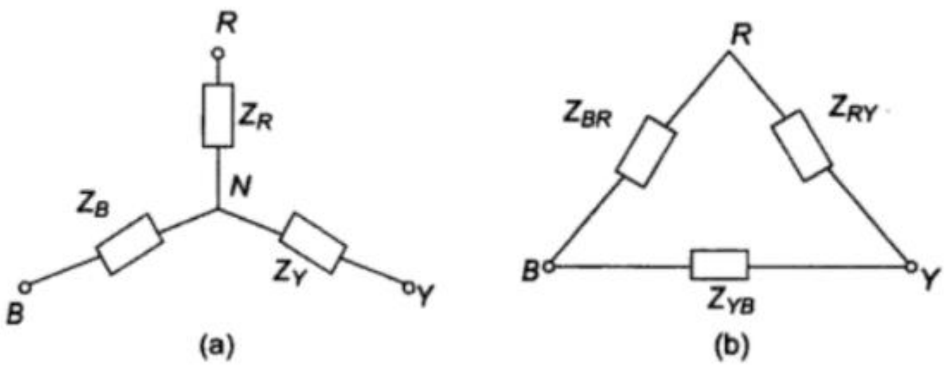


Note that the supply star point (Ns) is isolated from the load star point (N_L). The potential of the load star point is different from that of the supply star point. The result is that the load phase voltages are not equal to the supply phase voltage; and they are not only unequal in magnitude, but also subtend angles other than 120° with one another. The magnitude of each phase voltage depends upon the individual phase loads. The potential of the load neutral point changes according to changes in the impedances of the phases that is why sometimes the load neutral is also called a floating neutral point. All star-connected, unbalanced loads supplied from poly-phase systems without a neutral wire have floating neutral point; the phasor sum of the three unbalanced line currents is zero. The phase voltage of the load is not $1/\sqrt{3}$ of the line voltage. The unbalanced three-wire star load is difficult to deal with. It is because load phase voltages cannot be determined directly from the given

supply line voltages. There are many methods to solve such unbalanced Y-connected loads. Two frequently used methods are presented here. They are
(i) Star-delta conversion method, and
(ii) Millman's theorem method
(iii) Loop or Mesh Method

STAR-DELTA (or) DELTA-STAR CONVERSION METHOD

While dealing with currents and voltages in loads, it is often necessary to convert a star load to delta load and vice-versa. The conversion formulae (like as using resistances) can be applied even if the loads are unbalanced. Thus, considering Fig.(a), star load can be replaced by an equivalent delta-load with branch impedances as shown.



Delta impedances, in terms of star impedances, are

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

Consider Fig.(b), delta load can be replaced by an equivalent star-load with branch impedances as shown.

$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

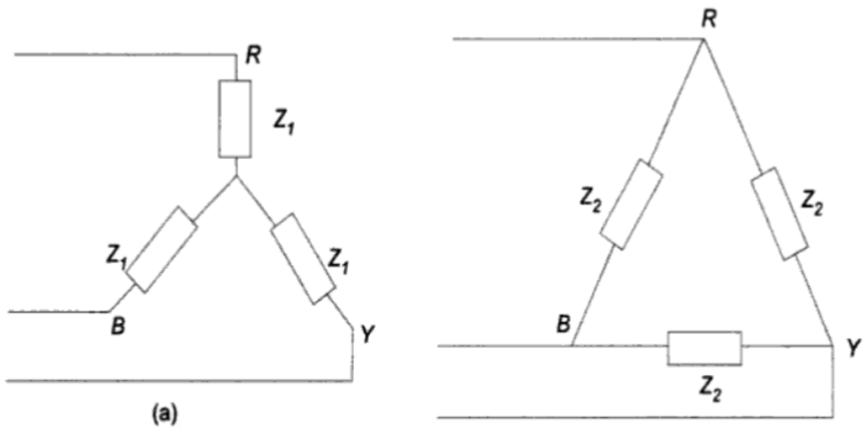
$$Z_Y = \frac{Z_{RY} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_B = \frac{Z_{BR} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

If the three-phase load is balanced, the balanced star-connected load having an impedance Z_1 in each phase as shown in the fig. The equivalent delta-connected load have an impedance of Z_2 in each phase as given by

$$Z_2 = 3Z_1$$

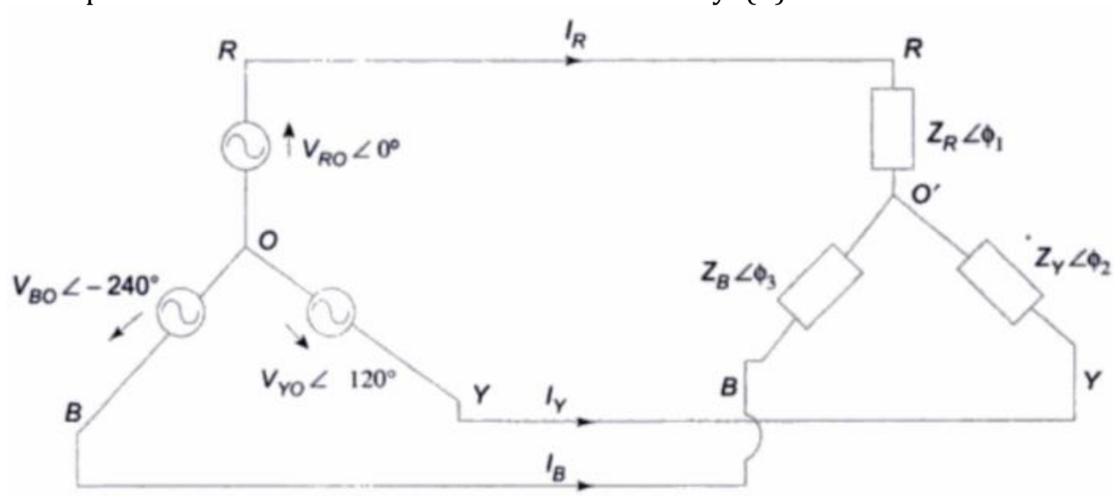
The star impedance in-terms of delta as $Z_1 = Z_2/3$



(a) (b)

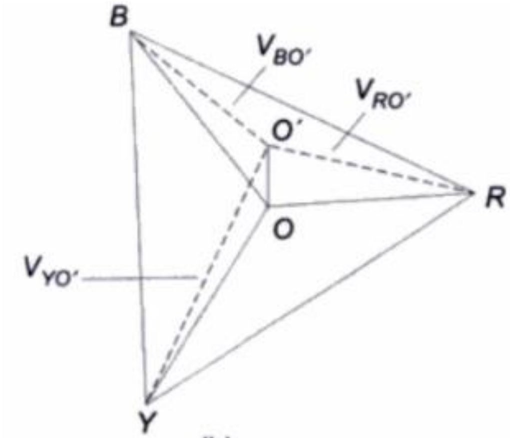
MILLMAN’S METHOD OF SOLVING UNBALANCED LOAD

Consider an unbalanced wye (Y) load connected to a balanced three-phase supply as shown in Fig.(a). V_{RO}, V_{YO} and V_{BO} are the phase voltages of the supply. They are equal in magnitude, but displaced by 120^0 from one another. $V_{RO'}$, $V_{YO'}$ and $V_{BO'}$, are the load phase voltages; they are unequal in magnitude as well as differ in phase by unequal angles. Z_R, Z_Y and Z_B are the impedances of the branches of the unbalanced wye(Y) connected load.



(a)

The Figure(b) shows the triangular phasor diagram of the complete system. Distances RY, YB and BR represent the line voltages of the supply as well as load. They are equal in magnitude, but displaced by 120^0. Here ‘O’ is the star-point of the supply and is located at the centre of the equilateral triangle RYB. O’ is the load star point. The star point of the supply which is at the zero potential is different from that of the star point at the load, due to the load being unbalanced. O’ has some potential with respect to O and is shifted away from the centre of the triangle. Distance O'O represents the voltage of the load star point with respect to the star point of the supply $V_{O'O}$.



(b)

$V_{o'o}$ is calculated using Millman's theorem. If $V_{o'o}$ is known, the load phase voltages and corresponding currents in the unbalanced wye load can be easily determined.

According to Millman's theorem, $V_{o'o}$ is given by

$$V_{o'o} = \frac{V_{Ro}\, Y_R + V_{Yo}\, Y_Y + V_{Bo}\, Y_B}{Y_R + Y_Y + Y_B}$$

where the parameters Y_R, Y_Y and Y_B are the admittances of the branches of the unbalanced wye connected load. From Fig.(a), we can write the equation

$$V_{Ro} = V_{Ro'} + V_{o'o}$$

or the load phase voltage

$$V_{Ro'} = V_{Ro} - V_{o'o}$$

Similarly, $V_{Yo'} = V_{Yo} - V_{o'o}$ and $V_{Bo'} = V_{Bo} - V_{o'o}$ can be calculated. The line currents in the load are

$$I_R = \frac{V_{Ro'}}{Z_R} = (V_{Ro} - V_{o'o})\, Y_R$$

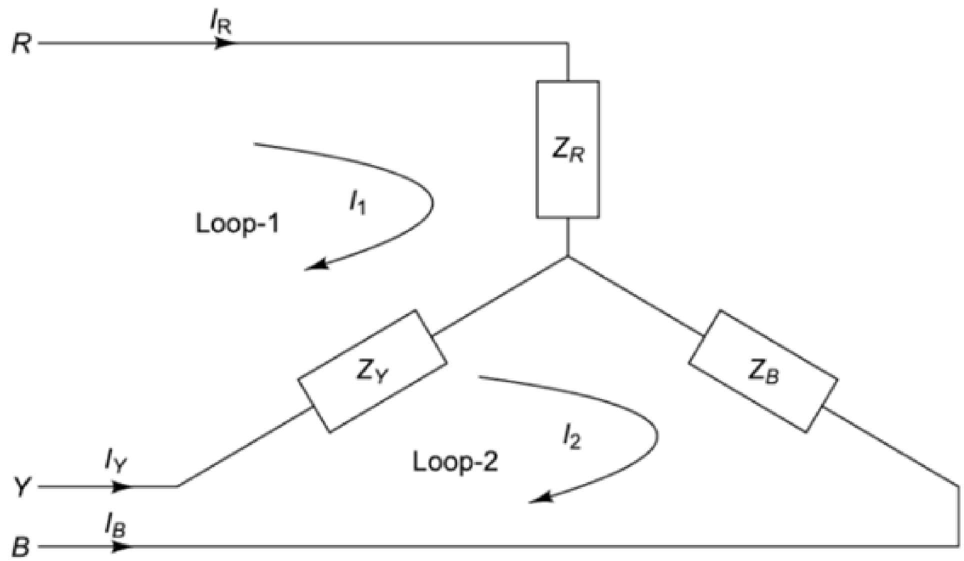
$$I_Y = \frac{V_{Yo'}}{Z_Y} = (V_{Yo} - V_{o'o})\, Y_Y$$

$$I_B = \frac{V_{Bo'}}{Z_B} = (V_{Bo} - V_{o'o})\, Y_B$$

[**NOTE:**
The unbalanced three-wire star-connected loads can also be determined by using Kirchhoff's laws and Maxwell's mesh or loop equation.]

LOOP (OR) MESH METHOD OF ANALYSIS

Consider an unbalanced three wire star connected load supplied by a balanced source with a phase sequence of RYB as shown in Fig. The three line currents are I_R, I_Y and I_B with the directions as indicated in the figure. The line currents can be obtained by mesh current method.



Assuming two loop currents I_1, I_2 in clockwise direction, we have the following equations.

Applying KVL for the 1st and 2nd loops

$$V_{RY} = I_1 Z_R + (I_1 - I_2) Z_Y$$
$$V_{YB} = (I_2 - I_1) Z_Y + I_2 Z_B$$

Where V_{RY} and V_{YB} are line voltages from balanced three phase supply

$$V_{RY} = V\angle 0^\circ$$
$$V_{YB} = V\angle -120^\circ$$
$$V_{BR} = V\angle -240^\circ$$

V is the magnitude of line voltage

Above equations can be written in matrix form as

$$\begin{bmatrix} Z_R + Z_Y & -Z_Y \\ -Z_Y & Z_B + Z_Y \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{RY} \\ V_{YB} \end{bmatrix}$$

By using crammers rule we can form

$$\Delta = \begin{bmatrix} Z_R + Z_Y & -Z_Y \\ -Z_Y & Z_B + Z_Y \end{bmatrix} \quad \Delta_1 = \begin{bmatrix} V_{RY} & -Z_Y \\ V_{YB} & Z_B + Z_Y \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} Z_R + Z_Y & V_{RY} \\ -Z_Y & V_{YB} \end{bmatrix}$$

From which we can find

$$I_1 = \frac{\Delta_1}{\Delta}, I_2 = \frac{\Delta_2}{\Delta}$$

The line currents which are also the phase currents I_R, I_Y and I_B are calculated from the mesh currents I_1 and I_2

$$I_R = I_1$$
$$I_Y = I_2 - I_1$$
$$I_B = -I_2$$

If Z_R, Z_Y and Z_B are phase impedances, the phase voltages are obtained by multipying the phase currents with the respective phase impedances.

Thus

$$V_R = V_{RN} = I_R Z_R$$
$$V_Y = V_{YN} = I_Y Z_Y$$
$$V_B = V_{BN} = I_B Z_B$$

Total power consumed in all three phases is given by

$$= |V_R|\,|I_R|\cos\phi_R + |V_Y|\,|I_Y|\cos\phi_Y + |V_B|\,|I_B|\cos\phi_B$$

where ϕ_R is the phase angle between V_R and I_R
where ϕ_Y is the phase angle between V_Y and I_Y
where ϕ_B is the phase angle between V_B and I_B

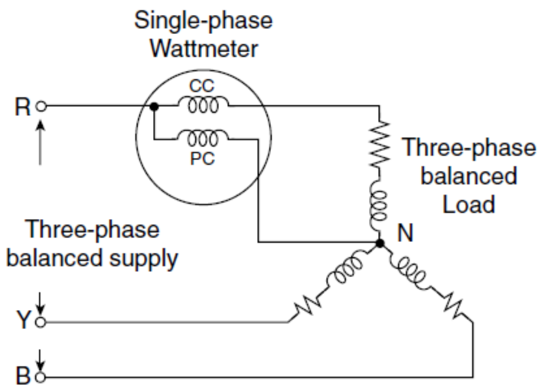
MEASUREMENT OF POWER IN THREE-PHASE CIRCUITS

In a three-phase system, total power is the sum of powers in three phases. The power is measured by wattmeter. It consists of two coils: (i) Current coil, and (ii) Voltage coil. Current coil is connected in series with the load and it senses current. Voltage coil is connected across supply terminals and it senses voltages.
There are three methods to measure three-phase power:
1. One-wattmeter method 2. Two-wattmeter method 3. Three-wattmeter method

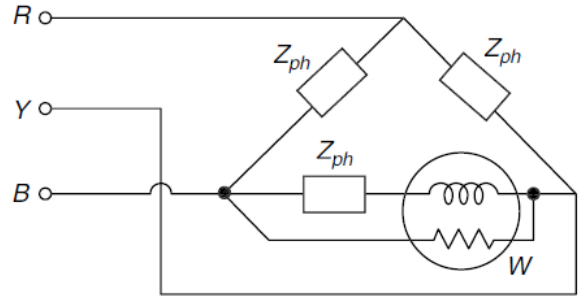
ONE-WATTMETER METHOD

In this method, only one single-phase wattmeter can be used to measure the total three-phase power. In this method, the current coil (CC) of the wattmeter is connected in series with any phase and the pressure coil (PC) is connected between that phase and the neutral, as shown in Figure. One-wattmeter method has a demerit that even a slight degree of unbalance in the load produces a large error in the measurement. In this method, one wattmeter will measure only the power of one phase. Hence, the total power is taken as three times the wattmeter reading.

$$\text{Total power} = 3 \times V_{Ph} I_{Ph} \cos\Phi$$



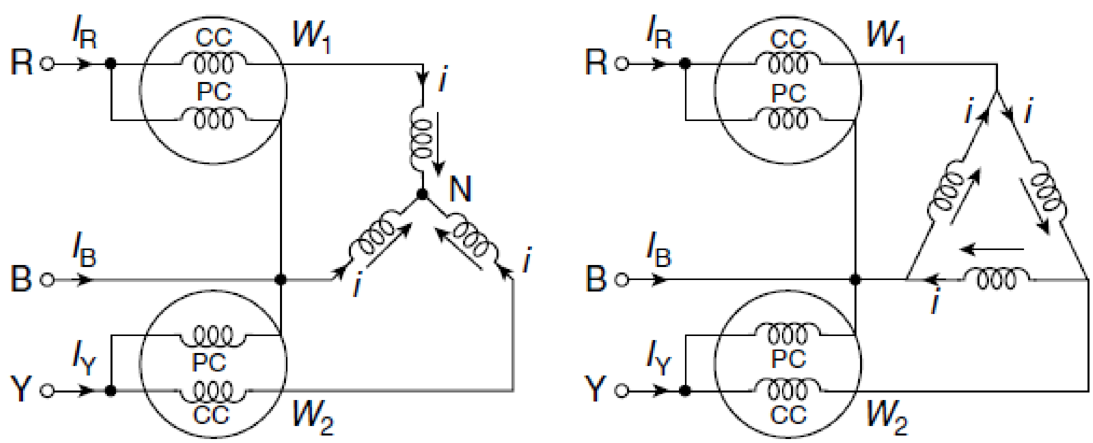
For Star-Load



For Delta-Load

TWO-WATTMETER METHOD

This method requires only two wattmeters to measure three-phase load for balanced as well as unbalanced loads. In this method, two wattmeters are connected in two phases and their pressure coils are connected to the remaining third phase, as shown in Figure.



This method of measurement is useful for balanced and unbalanced loads.

Let us consider the measurement of three-phase power of a star-connected load using two single-phase wattmeters as shown in Figure(a). We will calculate the power measured by the two wattmeters separately. Let W_1 and W_2, respectively, are the two-wattmeter readings.

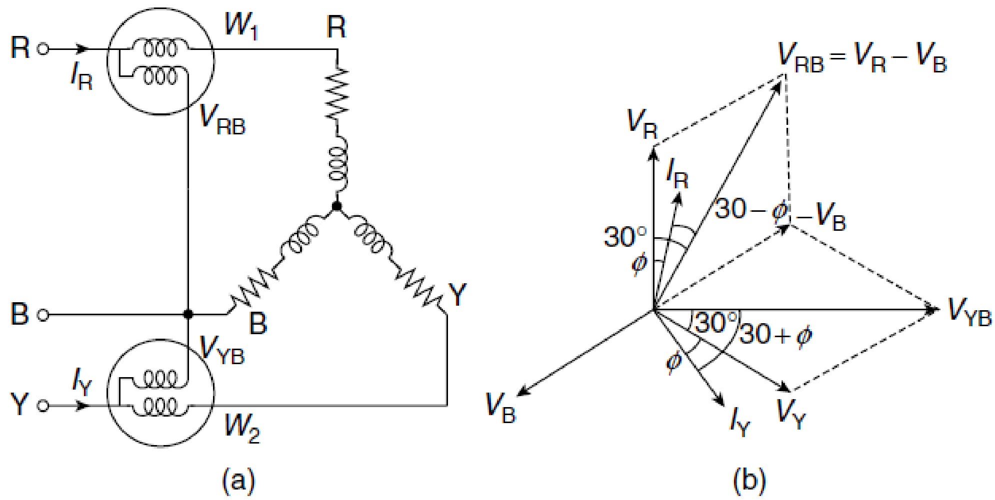
The current flowing through the current coil of wattmeter W_1 is I_R.

The voltage appearing across its pressure coil is V_{RB}.

The wattmeter reading will be equal to $W_1 = V_{RB} I_R$ cos of angle between V_{RB} and I_R.

Similarly, the wattmeter reading W_2 will be equal to $W_2 = V_{YB} I_Y$ cos of angle between V_{YB} and I_Y.

We will now draw the phasor diagram, and calculate W_1 and W_2.



From the phasor diagram, as shown in Figure(b), we get the equation as follows:

$$W_1 = V_{RB} I_R \cos(30-\phi) = \sqrt{3}\, V_{Ph} I_L \cos(30-\phi) = V_L I_L \cos(30-\phi)$$

$$W_2 = V_{YB} I_Y \cos(30+\phi) = \sqrt{3} V_{Ph} I_L \cos(30+\phi) = V_L I_L \cos(30+\phi)$$

We know that the total power in a three-phase circuit is $3V_{Ph} I_{Ph} \cos\phi$ or equal to $\sqrt{3}\, V_L I_L \cos\phi$.

Let us add the two wattmeter readings, that is, W_1 and W_2.

$$= W_1 + W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30-\phi) + \sqrt{3}\, V_{Ph} I_{Ph} \cos(30+\phi)$$

$$= \sqrt{3} V_{Ph} I_{Ph} [\cos(30-\phi) + \cos(30+\phi)]$$

$$= \sqrt{3}\, V_{Ph} I_{Ph}\, 2\cos\phi \cos 30^\circ$$

$$= \sqrt{3}\, V_{Ph} I_{Ph}\, 2\cos\phi \frac{\sqrt{3}}{2}$$

$$= 3V_{Ph} I_{Ph} \cos\phi$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

Thus, it is proved that the sum of the wattmeter readings is equal to the three-phase power.

Now, when the two wattmeter readings are subtracted from each other, we obtain the following form:

$$W_1 - W_2 = \sqrt{3}\, V_{Ph} I_{Ph} [\cos(30° - \phi) - \cos(30° + \phi)$$
$$= \sqrt{3}\, V_{Ph} I_{Ph}\, 2 \sin\phi \sin 30°$$

$$\sqrt{3}(W_1 - W_2) = 3 V_{Ph} I_{Ph} \sin\phi$$

$$\sqrt{3}\,(W_1 - W_2) = \sqrt{3}\; V_L I_L \sin\phi$$

by Dividing equation

$$\frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\, V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi} = \tan\phi$$

$$\phi = \tan^{-1} \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$$

Power factor $\cos\phi = \cos\left[\tan^{-1} \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$

Note:

For Leading Power factor $\cos\phi = \cos\left[\tan^{-1}\left(-\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right)\right]$

EFFECT OF CHANGE IN POWER FACTOR ON WATTMETER READINGS

Let the wattmeter readings as follows:

$$W_1 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 - \phi)$$
$$W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 + \phi)$$

i) At unity power factor, when $\cos\phi = 1$, that is, $\phi = 0$

$$W_1 = \sqrt{3}\, V_{Ph} I_{Ph} \cos 30°$$
$$W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos 30°$$

Thus, at power factor = 1, both the wattmeter readings will be positive and of equal value.

ii) At 0.5 power factor, $\cos\phi = 0.5$, that is, $\phi = 60^\circ$

$$W_1 = \sqrt{3} V_{Ph} I_{Ph} \cos(-30^\circ)$$
$$= \sqrt{3}\, V_{Ph} I_{Ph} \cos 30^\circ$$
$$W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30^\circ + 60^\circ) = 0$$

Thus, at power factor equal to 0.5, one of the wattmeters will give zero reading.

iii) When the power factor is less than 0.5, that is, $\phi > 60$. Let us observe the wattmeter readings.

$$W_1 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 - \phi)$$
$$W_2 = \sqrt{3}\, V_{Ph} I_{Ph} \cos(30 + \phi)$$

When $\phi > 60$, W_1 will give positive readings but W_2 will give a negative reading. Thus, for power factor less than 0.5, that is, for $\phi > 60^\circ$, one of the wattmeters will give a negative reading.

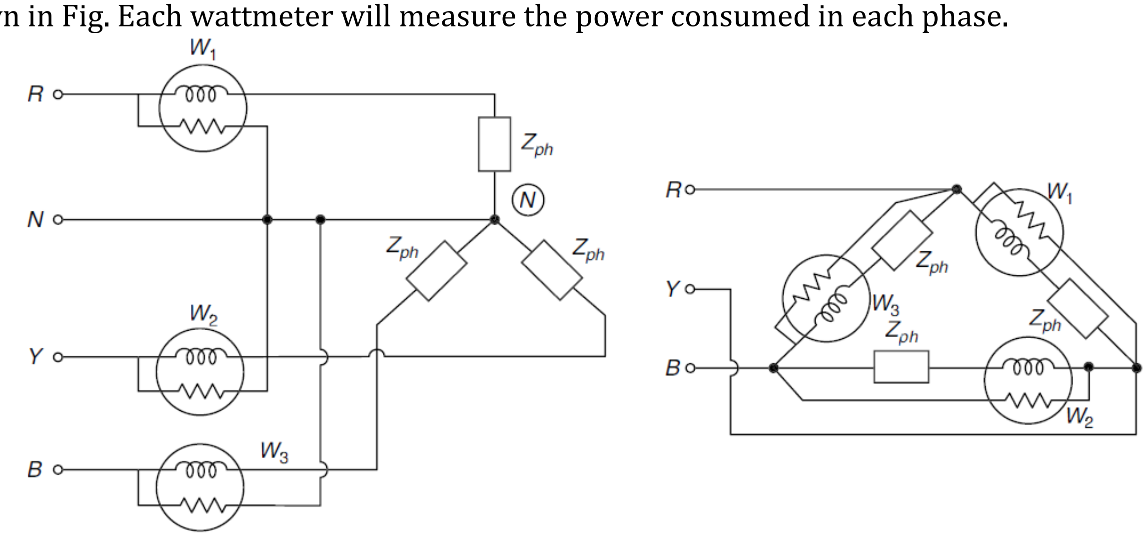
iv) When the load is purely inductive or capacitive, the power factor will be zero, that is, $\phi = 90^\circ$

$$W_1 = V_L I_L \cos(30^\circ - 90^\circ) = V_L I_L \cos 60^\circ$$
$$W_2 = V_L I_L \cos(30^\circ + 90^\circ) = -V_L I_L \sin 30^\circ$$

Both the wattmeters show equal but opposite readings. Hence, the total power consumed will be zero.

THREE-WATTMETER METHOD

This method is used for balanced as well as unbalanced loads. Three wattmeters are inserted in each of the three phases of the load whether star connected or delta connected as shown in Fig. Each wattmeter will measure the power consumed in each phase.



For Balanced load, $W_1 = W_2 = W_3$

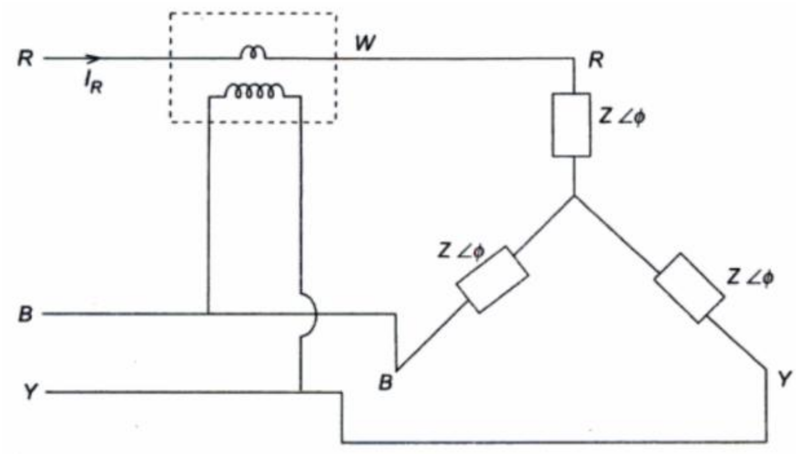
For Un-balanced load, $W_1 \neq W_2 \neq W_3$

Total power, $P = W_1 + W_2 + W_3$

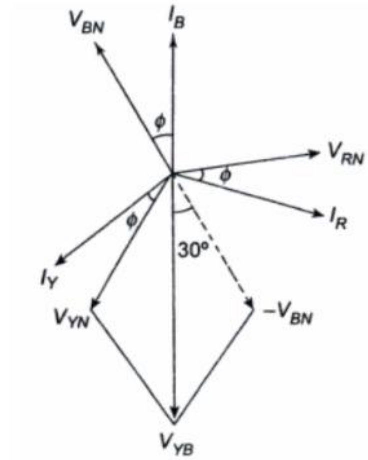
MEASUREMENT OF REACTIVE POWER

The total reactive power = $\sqrt{3}\, V_L I_L \sin\Phi$. Reactive power in a balanced three-phase load can also be calculated by using a single wattmeter.

As shown in Fig.(a), the current coil of the wattmeters is connected in any one line (R in this case), and the pressure coil across the other two lines (between Y and B in this case). Assuming phase sequence RYB and an inductive load of angle Φ, the phasor diagram for the circuit is shown in Fig.(b).



(a)



(b)

From Fig.(a), it is clear that the wattmeter power is proportional to the product of current through its current coil I_R. voltage across its pressure coil, V_{YB}, and cosine of the angle between V_{YB} and I_R.

$$V_{YB} = V_{YN} - V_{BN} = V_L$$

From the vector diagram the angle between V_{YB} and I_R is (90 - Φ)°

$$\therefore \quad \text{Wattmeter reading} = V_{YB} I_R \cos(90 - \phi)^\circ$$
$$= V_L I_L \sin\phi \text{ VAR}$$

If the above expression is multiplied by √3, we get the total reactive power in the load.

SOLVED PROBLEMS

1) Three equal impedances, each of (8 + j10) ohms, are connected in star. This is further connected to a 440 V, 50 Hz, three-phase supply. Calculate (a) phase voltage, (b) phase angle, (c) phase current, (d) line current, (e) active power, and (f) reactive power.

SOL:

$$\mathbf{Z}_{ph} = (8 + j10)\,\Omega, \quad V_L = 440\text{ V}, \quad f = 50\text{ Hz}$$

For a star-connected load,

(a) Phase voltage
$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03\text{ V}$$

(b) Phase angle
$$\mathbf{Z}_{ph} = 8 + j10 = 12.81\angle 51.34^\circ\,\Omega$$
$$Z_{ph} = 12.81\,\Omega$$
$$\phi = 51.34^\circ$$

(c) Phase current
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{12.81} = 19.83\text{ A}$$

(d) Line current $I_L = I_{ph} = 19.83\text{ A}$

(e) Active power $P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 440 \times 19.83 \times \cos(51.34^\circ) = 9.44\text{ kW}$

(f) Reactive power $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3} \times 440 \times 19.83 \times \sin(51.34^\circ) = 11.81\text{ kVAR}$

2) Three similar coils A,B, and C are available. Each coil has a 9 Ω resistance and a 12 Ω reactance. They are connected in delta to a three-phase, 440 V, 50 Hz supply. Calculate for this load, the (a) phase current, (b) line current, (c) power factor, (d) total kVA, (e) active power, and (f) reactive power. If these coils are connected in star across the same supply, calculate all the above quantities.

SOL:

$$R = 9\,\Omega, \quad X_L = 12\,\Omega, \quad V_L = 440\text{ V}, \quad f = 50\text{ Hz}$$

For a delta-connected load,

(a) Phase current
$$V_L = V_{ph} = 440\text{ V}$$
$$\mathbf{Z}_{ph} = R + jX_L = 9 + j12 = 15\angle 53.13^\circ\,\Omega$$
$$Z_{ph} = 15\,\Omega$$
$$\phi = 53.13^\circ$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{15} = 29.33\text{ A}$$

(b) Line current $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 29.33 = 50.8$ A

(c) Power factor pf $= \cos\phi = \cos(53.13°) = 0.6$ (lagging)

(d) Total kVA $S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 440 \times 50.8 = 38.71$ kVA

(e) Active power $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 440 \times 50.8 \times 0.6 = 23.23$ kW

(f) Reactive power $Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 440 \times 50.8 \times \sin(53.13°) = 30.97$ kVAR

If these coils are connected in star across the same supply,

(a) Phase current

$$V_L = 440 \text{ V}$$

$$Z_{ph} = 15\ \Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{15} = 16.94 \text{ A}$$

(b) Line current $I_L = I_{ph} = 16.94$ A

(c) Power factor pf $= 0.6$ (lagging)

(d) Total kVA $S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 440 \times 16.94 = 12.91$ kVA

(e) Active power $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 440 \times 16.94 \times 0.6 = 7.74$ kW

(f) Reactive power $Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 440 \times 16.94 \times \sin(53.13°) = 12.33$ kVAR

3) A 415 V, 50 Hz, three-phase voltage is applied to three star-connected identical impedances. Each impedance consists of a resistance of 15Ω, a capacitance of 177μF and an inductance of 0.1H in series. Find the (a) power factor, (b) phase current, (c) line current, (d) active power, (e) reactive power, and (f) total VA. Draw a neat phasor diagram. If the same impedances are connected in delta, find the (a) line current, and (b) power consumed.

SOL:

$$V_L = 415 \text{ V}, \quad f = 50 \text{ Hz}, \quad R = 15\ \Omega, \quad C = 177\ \mu\text{F}, \quad L = 0.1 \text{ H}$$

For a star-connected load,

(a) Power factor $X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 177 \times 10^{-6}} = 17.98\ \Omega$$

$$\mathbf{Z}_{ph} = R + jX_L - jX_C = 15 + j31.42 - j17.98 = 15 + j13.44 = 20.14\ \angle 41.86°\ \Omega$$

$$Z_{ph} = 20.14\ \Omega$$

$$\phi = 41.86°$$

$$\text{pf} = \cos\phi = \cos(41.86°) = 0.744 \text{ (lagging)}$$

(b) Phase current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{20.14} = 11.9 \text{ A}$$

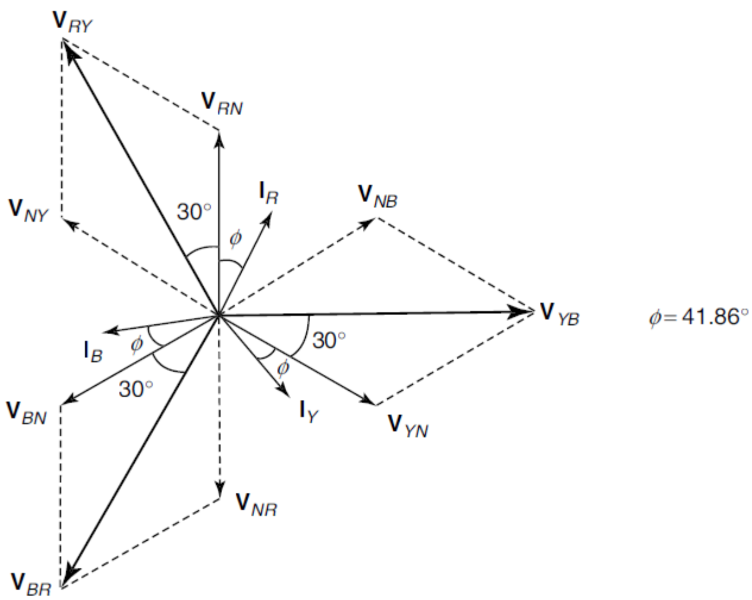
(c) Line current $I_L = I_{ph} = 11.9$ A

(d) Active power $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 415 \times 11.9 \times 0.744 = 6.36$ kW

(e) Reactive power $Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 415 \times 11.9 \times \sin(41.86°) = 5.71$ kVAR

(f) Total VA $S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 415 \times 11.9 = 8.55$ kVA

The phasor diagram is shown in Fig.



If the same impedances are connected in delta,

(a) Line current

$$V_L = V_{ph} = 415 \text{ V}$$

$$Z_{ph} = 20.14\ \Omega$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{20.14} = 20.61 \text{ A}$$

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 20.61 = 35.69 \text{ A}$$

(b) Power consumed $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 415 \times 35.69 \times 0.774 = 19.09$ kW

4) In the two-wattmeter method of power measurement for a three-phase load, the readings of the wattmeter are 1000 W and 550 W. What is the power factor of the load?

SOL:

$$W_1 = 1000\,\text{W},\ W_2 = 550\,\text{W}$$

Power factor of load

$$\cos\phi = \cos\tan^{-1}\sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}$$

$$= \cos\tan^{-1}\sqrt{3}\frac{1000 - 550}{1000 + 550}$$

$$\cos\phi = \cos\tan^{-1} 0.5 = 0.9$$

5) In the measurement of three-phase power by the two wattmeter method, for a certain load, one of the wattmeters reads 20 kW and the other 5 kW after the current coil connection of one of the wattmeters has been reversed. Calculate the power and power factor of the load.

SOL:

$$W_1 = 20\,\text{kW}$$

$$W_2 = -\,5\,\text{kW}$$

$$P = W_1 + W_2 = 20 - 5 = 15\,\text{kW}$$

Power factor of the load $= \cos\tan^{-1}\dfrac{W_1 - W_2}{W_1 + W_2}\sqrt{3}$

$$= \cos\tan^{-1}\frac{20-(-5)}{20+(-5)}\sqrt{3}$$

$$= 0.3273 \text{ lagging}$$

6) In two wattmeter measurement, the load connected was 30 kW at 0.7 pf lagging. Find the reading of each wattmeter.

SOL:

We know that the reading of the two wattmeters will be $\sqrt{3}V_{\text{Ph}}I_{\text{Ph}}\cos(30-\phi)$ and $\sqrt{3}\,V_{\text{Ph}}I_{\text{Ph}}\cos(30+\phi)$, respectively.

For star connection, $\sqrt{3}\,V_{\text{Ph}} = V_{\text{L}}$ and $I_{\text{Ph}} = I_{\text{L}}$

The total load $P = 30\,\text{kW}$

Power factor $\cos\phi = 0.7$ lagging

Phase angle $\phi = \cos^{-1}(0.7) = 45.57°$ lagging

$$V_L I_L = \frac{P \text{ in kW} \times 1000}{\sqrt{3}\cos\phi}$$

$$= \frac{30 \times 1000}{\sqrt{3} \times 0.7} = 24743.6 \text{ VA}$$

Reading of wattmeter $W_1 = V_L\, I_L \cos(30 - \phi)$

$= 24743.6 \cos(30 - 45.57°)$

$= 23.835\,\text{kW}$

Reading of wattmeter $W_2 = V_L\, I_L \cos(30 + \phi)$

$= 24743.6 \cos(30 + 45.57°)$

$= 6.165\,\text{kW}$

Thus, the total power is calculated as $P = W_1 + W_2 = 23.835 + 6.165$

$= 30\,\text{kW}$

7) Three equal impedances, each consisting of R and L in series are connected in star and are supplied from a 400 V, 50 Hz, three-phase, three-wire balanced supply system. The power input to the load is measured by the two-wattmeter method and the two wattmeters read 3 kW and 1 kW respectively. Determine the values of R and L connected in each phase.

SOL:

Reading of wattmeter 1 $W_1 = 3\,\text{kW}$

Reading of wattmeter 2 $W_2 = 1\,\text{kW}$

Total power $P = W_1 + W_2 = 3 + 1 = 4\,\text{kW}$

Power factor of the circuit, $\cos\phi = \cos\tan^{-1}\frac{W_1 - W_2}{W_1 - W_2}\sqrt{3}$

$$= \cos\tan^{-1}\frac{3-1}{3+1}\sqrt{3}$$

$= \cos 40.89$

$= 0.7559$ lagging

Line current $I_L = \frac{P}{\sqrt{3}\, V_L \cos\phi}$

$$= \frac{4 \times 1000}{\sqrt{3} \times 400 \times 0.7559} = 7.64 \text{ A} = \text{Phase current } I_P$$

Impedance of the circuit per phase $Z = \frac{V_P}{I_P} = \frac{400\sqrt{3}}{7.64}$

$$= 30.237\,\Omega$$

$$R = Z\cos\phi = 30.237 \times 0.7559 = 22.856\,\Omega$$

Reactance per phase $X_L = \sqrt{Z^2 - R^2}$

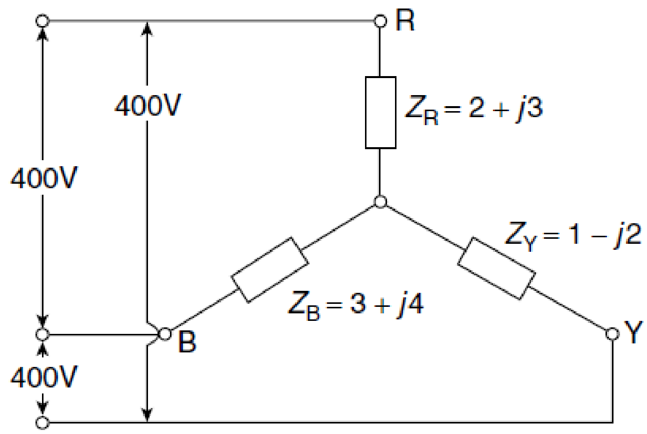
$$= \sqrt{(30.237)^2 - (22.856)^2}$$

$$= 19.796\,\Omega$$

Inductance per phase $L = \frac{X_L}{2\pi f}$

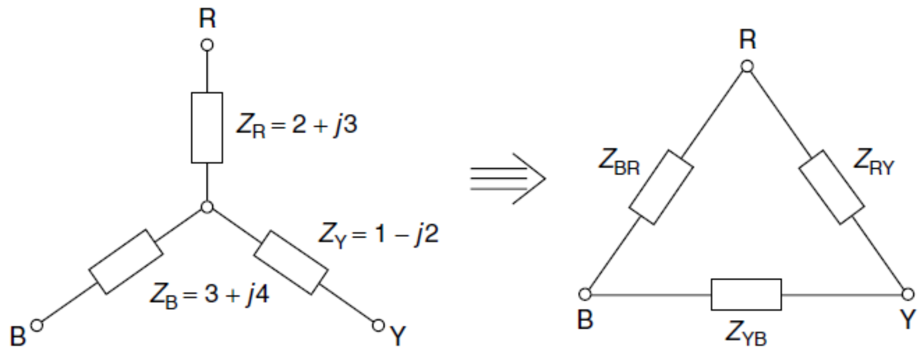
$$= \frac{19.796}{2\pi \times 50} = 0.063\text{ H} = 63\text{ mH}$$

8) A three-phase 400 V, 50 Hz supply is connected across a three-phase load as shown in Figure. Calculate the equivalent delta load.



SOL:

The equivalent delta load is shown by assuming clockwise phase sequence, that is, phase sequence of RYB as shown in Figure are calculated as



$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_r}{Z_B}$$

$$= 2 + j3 + 1 - j3 + \frac{(2+j3)(1-j3)}{3+j4}$$

$$= 3 + \frac{\sqrt{2^2+3^2}\angle\tan^{-1}\frac{3}{2} \times \sqrt{1^2+3^2}\angle -\tan^{-1}\frac{3}{1}}{\sqrt{3^2+4^2}\angle\tan^{-1}\frac{4}{3}}$$

$$= 3 + \frac{3.61\ \angle 56.3^\circ \times 3.162\ \angle -66^\circ}{5\ \angle 53.13^\circ}$$

$$= 3 + 2.23\angle -62.83^\circ$$

$$= 3 + 2.23[\cos 62.83^\circ - j\sin 62.83]$$

$$= 4.02 - j\ 1.98 = 4.48\angle -26^\circ$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}$$

$$= 1 - j3 + 3 + j4 + \frac{(1-j3)(3+j4)}{2+j3}$$

$$= 4 + j1 + \frac{\sqrt{1^2+3^2}\ \angle -\tan^{-1}\frac{1}{3} \times \sqrt{3^2+4^2}\ \angle\tan^{-1}\frac{4}{3}}{\sqrt{2^2+3^2}\ \angle\tan^{-1}\frac{3}{2}}$$

$$= 4 + j1 + \frac{3.162\ \angle -66^\circ \times 5\ \angle 53.13^\circ}{3.6\ \angle 58^\circ}$$

$$= 4 + j1 + 4.39\angle -66^\circ + 53.13^\circ - 58^\circ$$

$$= 4 + j1 + 4.93\ \angle -71.87^\circ$$

$$= 4 + j1 + 4.93(\cos 71.87^\circ - j\ 71 - 87^\circ).$$

$$= 5.53 - j353.3 = 353.4\angle -89^\circ$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}$$

$$= 3 + j4 + 2 + j3 + \frac{(3+j4)(2+j3)}{1-j3}$$

$$= 5 + j7 + \frac{5\angle 53.13^\circ \times 3.61\angle 56.3^\circ}{3.162\angle -66^\circ}$$

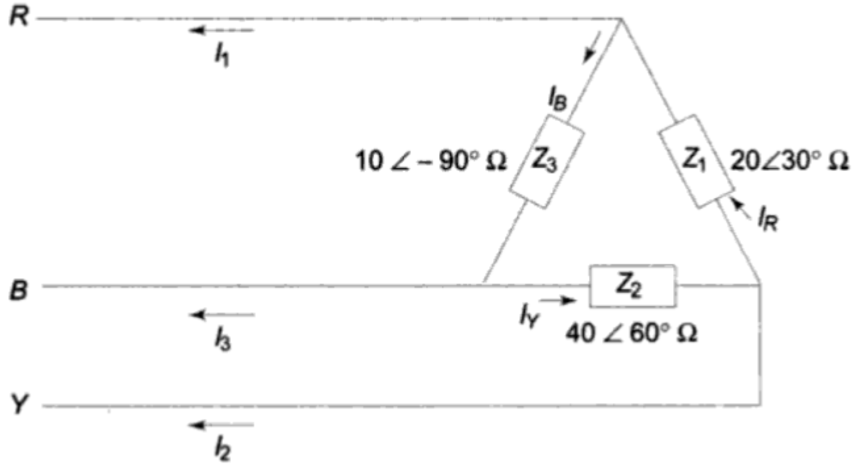
$$= 5 + j7 + \frac{18.05\ \angle 109.43}{3.162\angle -66^\circ}$$

$$= 5 + j7 + 5.72\ \angle 175.43^\circ$$

$$= 5 + j7 + 5.73(\cos 175.43^\circ + j\sin 175.43^\circ)$$

$$= -0.707 + j7.46 = 7.49\angle 95.4^\circ$$

9) Three impedances $Z_1=20\angle30^0\Omega$, $Z_2=40\angle60^0\ \Omega$ and $Z_3=10\angle-90^0\ \Omega$ are delta connected to a 400V, 3-phase system as shown in fig. Determine a) Phase currents b) line currents and c) total power consumed by the load.



SOL:

The three phase currents are I_R, I_Y and I_B, and the three line currents are I_1, I_2 and I_3. Taking $V_{RY} = V\angle 0°$ V as reference phasor, and assuming *RYB* phase sequence, we have

$$V_{RY} = 400\angle 0°\text{ V},\ V_{YB} = 400\angle -120°\text{ V},$$

$$V_{BR} = 400\angle -240°\text{ V}$$

$$Z_1 = 20\angle 30°\ \Omega = (17.32 + j10)\ \Omega;$$

$$Z_2 = 40\angle 60°\ \Omega = (20 + j34.64)\ \Omega;$$

$$Z_3 = 10\angle -90°\ \Omega = (0 - j10)\ \Omega$$

$$I_R = \frac{V_{RY}}{Z_1\angle\phi_1} = \frac{400\angle 0°}{20\angle 30°}\text{ A} = 20\angle -30°\text{ A} = (17.32 - j10)\text{ A}$$

$$I_Y = \frac{V_{YB}}{Z_2\angle\phi_2} = \frac{400\angle -120°}{40\angle 60°}\text{ A} = 10\angle -180°\text{ A} = (-10 + j0)\text{ A}$$

$$I_B = \frac{V_{BR}}{Z_3\angle\phi_3} = \frac{400\angle -240°}{10\angle -90°}\text{ A} = 40\angle -150°\text{ A} = (-34.64 - j20)\text{ A}$$

Now the three line currents are

$$I_1 = I_R - I_B = [(17.32 - j10) - (-34.64 - j20)]$$
$$= (51.96 + j10)\text{ A} = 52.91\angle 10.89°\text{ A}$$

$$I_2 = I_Y - I_R = [(-10 + j0) - (17.32 - j10)]$$
$$= (-27.32 + j10)\text{ A} = 29.09\angle 159.89°\text{ A}$$

$$I_3 = I_B - I_Y = [(-34.64 - j20) - (-10 + j0)]$$
$$= (-24.64 - j20)\text{ A} = 31.73\angle -140.94°\text{ A}$$

(iii) To calculate the total power, first the powers in the individual phases are to be calculated, then they are added up to get the total power in the unbalanced load.

$$\text{Power in } R \text{ phase} = I_R^2 \times R_R = (20)^2 \times 17.32 = 6928 \text{ W}$$

$$\text{Power in } Y \text{ phase} = I_Y^2 \times R_Y = (10)^2 \times 20 = 2000 \text{ W}$$

$$\text{Power in } B \text{ phase} = I_B^2 \times R_B = (40)^2 \times 0 = 0$$

$$\text{Total power in the load} = 6928 + 2000 = 8928 \text{ W}$$

10) An unbalanced four-wire, star connected load has a balanced voltage of 400V, the loads are

$Z_1=(4+j8)\Omega$, $Z_2=(3+j4)\Omega$, $Z_3=(15+j20)\Omega$

Calculate i) line currents ii) Current in the neutral wire iii) the total power

SOL:

$$Z_1 = (4 + j8)\ \Omega;\ Z_2 = (3 + j4)\ \Omega;\ Z_3 = (15 + j20)\ \Omega$$

$$Z_1 = 8.94\ \angle 63.40^\circ\ \Omega;\ Z_2 = 5\ \angle 53.1^\circ\ \Omega;\ Z_3 = 25\ \angle 53.13^\circ\ \Omega$$

Let us assume *RYB* phase sequence.

The phase voltage $V_{RN} = \dfrac{400}{\sqrt{3}} = 230.94$ V.

Taking V_{RN} as the reference phasor, we have

$$V_{RN} = 230.94\ \angle 0^\circ \text{ V},\ V_{YN} = 230.94\ \angle -120^\circ \text{ V}$$

$$V_{BN} = 230.94\ \angle -240^\circ \text{ V}$$

The three line currents are

(i) $I_R = \dfrac{V_{RN}}{Z_1} = \dfrac{230.94\ \angle 0^\circ}{8.94\ \angle 63.4^\circ} \text{ A} = 25.83\ \angle -63.4^\circ \text{ A}$

$$I_Y = \frac{V_{YN}}{Z_2} \quad \frac{230.94\angle -120}{5\angle 53.1} \text{ A} = 46.188\ \angle -173.1^\circ \text{ A}$$

$$I_B = \frac{V_{BN}}{Z_3} \quad \frac{230.94\angle -240}{25\angle 53.13} \text{ A} = 9.23\ \angle -293.13^\circ \text{ A}$$

(ii) To find the neutral current, we must add the three line currents. The neutral current must then be equal and opposite to this sum.

Thus, $I_N = -(I_R + I_Y + I_B)$

$$= -(25.83\ \angle -63.4^\circ + 46.188\ \angle -173.1^\circ + 9.23\ \angle -293.13^\circ) \text{ A}$$

$$I_N = -[(11.56 - j23.09) + (-45.85 - j5.54) + (3.62 + j8.48)] \text{ A}.$$

$$I_N = -[(-30.67 - j20.15)] \text{ A} = (30.67 + j20.15) \text{ A}$$

$$I_N = 36.69\ \angle 33.30^\circ \text{ A}$$

(iii) Power in R phase $= I_R^2 \times R_R = (25.83)^2 \times 4 = 2668.75$ W

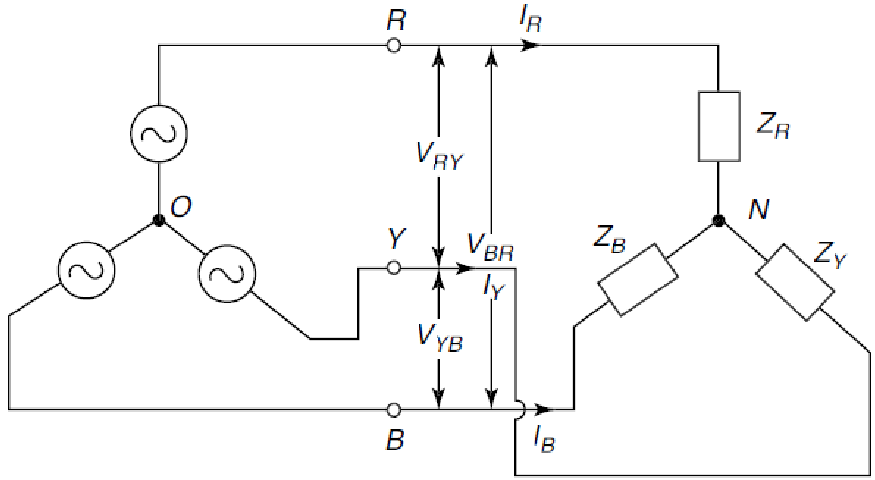
Power in Y phase $= I_Y^2 \times R_Y = (46.18)^2 \times 3 = 6397.77$ W

Power in B phase $= I_B^2 \times R_B = (9.23)^2 \times 15 = 1277.89$ W

Total power absorbed by the load

$$= 2668.75 + 6397.77 + 1277.89 = 10344.41 \text{ W}$$

11) A symmetrical 440 V, 3-phase system supplied a star-connected load with the branch impedances Z_R=10 Ω, Z_Y= j5Ω, Z_B=−j5Ω as shown in Fig. Calculate line currents and voltage across each phase impedance by Millman's theorem. The phase sequence is RYB.



SOL:

$$V_L = 440 \text{ V}, \quad \mathbf{Z}_R = 10\ \Omega, \quad \mathbf{Z}_Y = j5\ \Omega, \quad \mathbf{Z}_B = -j5\ \Omega$$

For a star-connected load,

$$V_{ph=} \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

Let O be the neutral point of the supply system.

$$\text{Let} \quad V_{RO} = 254.03\angle 0° \text{ V}$$

$$V_{YO} = 254.03\angle -120° \text{ V}$$

$$V_{BO} = 254.03\angle -240° \text{ V}$$

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{10} = \frac{1}{10\angle 0°} = 0.1\angle 0° \ \mho$$

$$\mathbf{Y}_Y = \frac{1}{\mathbf{Z}_Y} = \frac{1}{j5} = \frac{1}{5\angle 90°} = 0.2\angle -90° \ \mho$$

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{1}{-j5} = \frac{1}{5\angle -90°} = 0.2\angle 90° \ \mho$$

By Millman's theorem,

$$\mathbf{V}_{NO} = \frac{\mathbf{V}_{RO}\ \mathbf{Y}_R + \mathbf{V}_{YO}\ \mathbf{Y}_Y + \mathbf{Y}_{BO}\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

$$= \frac{(254.03\angle 0°)(0.1\angle 0°) + (254.03\angle -120°)(0.2\angle -90°) + (254.03\angle -240°)(0.2\angle 90°)}{0.1\angle 0° + 0.2\angle -90° + 0.2\angle 90°}$$

$$= 625.96\angle 180° \text{ V}$$

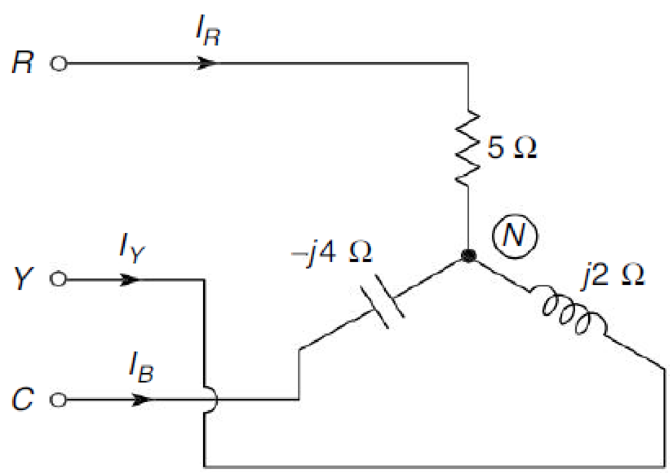
Voltage across phase impedance

$$\mathbf{V}_{RN} = \mathbf{V}_{RO} - \mathbf{V}_{NO} = 254.03\angle 0° - 625.96\angle 180° = 880\angle 0° \text{ V}$$
$$\mathbf{V}_{YN} = \mathbf{V}_{YO} - \mathbf{V}_{NO} = 254.03\angle -120° - 625.96\angle 180° = 545.29\angle -23.79° \text{ V}$$
$$\mathbf{V}_{BN} = \mathbf{V}_{BO} - \mathbf{V}_{NO} = 254.03\angle -240° - 625.96\angle 180° = 545.29\angle 23.79° \text{ V}$$

Phase currents/ line currents

$$\mathbf{I}_R = \frac{\mathbf{V}_{RN}}{\mathbf{Z}_R} = \frac{880\angle 0°}{10\angle 0°} = 88\angle 0° \text{ A}$$
$$\mathbf{I}_Y = \frac{\mathbf{V}_{YN}}{\mathbf{Z}_Y} = \frac{545.29\angle -23.79°}{5\angle 90°} = 109.06\angle -113.79° \text{ A}$$
$$\mathbf{I}_B = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B} = \frac{545.29\angle 23.79°}{5\angle -90°} = 109.06\angle 113.79° \text{ A}$$

12) In the circuit of Fig., a symmetrical 3-phase 100V, three-wire supply feeds an unbalanced star-connected load, with impedances of the load as $Z_R=5\angle 0^0 \Omega$, $Z_Y=2\angle 90^0\ \Omega$ and $Z_B=4\angle -90^0\ \Omega$. Find the line currents and voltage across the impedance using star-delta transformation method.



SOL:

$$V_L = 100 \text{ V}, \quad \mathbf{Z}_R = 5\angle 0°\ \Omega, \quad \mathbf{Z}_Y = 2\angle 90°\ \Omega, \quad \mathbf{Z}_B = 4\angle -90°\ \Omega$$

The unbalanced star-connected load can be converted into equivalent delta-connected load by star-delta transformation technique.

$$\mathbf{Z}_{RY} = \mathbf{Z}_R + \mathbf{Z}_Y + \frac{\mathbf{Z}_R\mathbf{Z}_Y}{\mathbf{Z}_B} = 5\angle 0° + 2\angle 90° + \frac{(5\angle 0°)(2\angle 90°)}{4\angle -90°} = 3.2\angle 38.66°\ \Omega$$

$$\mathbf{Z}_{YB} = \mathbf{Z}_Y + \mathbf{Z}_B + \frac{\mathbf{Z}_Y\mathbf{Z}_B}{\mathbf{Z}_R} = 2\angle 90° + 4\angle -90° + \frac{(2\angle 90°)(4\angle -90°)}{5\angle 0°} = 2.56\angle -51.34°\ \Omega$$

$$\mathbf{Z}_{BR} = \mathbf{Z}_B + \mathbf{Z}_R + \frac{\mathbf{Z}_B\mathbf{Z}_R}{\mathbf{Z}_Y} = 4\angle -90° + 5\angle 0° + \frac{(4\angle -90°)(5\angle 0°)}{2\angle 90°} = 6.4\angle -141.34°\ \Omega$$

The equivalent delta-connected load is shown in Fig.

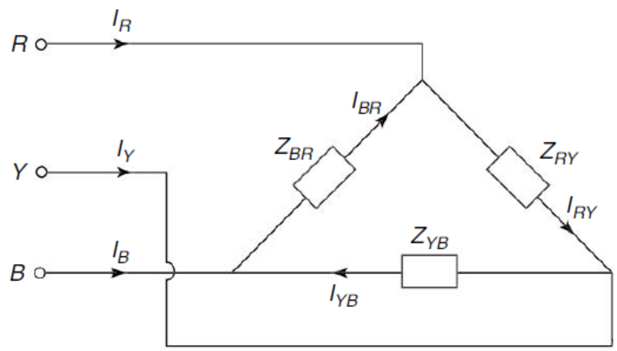
For a delta-connected load,

$$V_{ph} = V_L = 100\text{ V}$$

$$\text{Let}\quad \mathbf{V}_{RY} = 100\angle 0°\text{ V}$$

$$\mathbf{V}_{YB} = 100\angle -120°\text{ V}$$

$$\mathbf{V}_{BR} = 100\angle -240°\text{ V}$$



Phase currents

$$\mathbf{I}_{RY} = \frac{\mathbf{V}_{RY}}{\mathbf{Z}_{RY}} = \frac{100\angle 0°}{3.2\angle 38.66°} = 31.25\angle -38.66°\text{ A}$$

$$\mathbf{I}_{YB} = \frac{\mathbf{V}_{YB}}{\mathbf{Z}_{YB}} = \frac{100\angle -120°}{2.56\angle -51.34°} = 39.06\angle -68.66°\text{ A}$$

$$\mathbf{I}_{BR} = \frac{\mathbf{V}_{BR}}{\mathbf{Z}_{BR}} = \frac{100\angle -240°}{6.4\angle -141.34°} = 15.63\angle -98.66°\text{ A}$$

Line currents

$$\mathbf{I}_R = \mathbf{I}_{RY} - \mathbf{I}_{BR} = 31.25\angle -38.66° - 15.63\angle -98.66° = 27.06\angle -8.65°\text{ A}$$

$$\mathbf{I}_Y = \mathbf{I}_{YB} - \mathbf{I}_{RY} = 39.06\angle -68.66° - 31.25\angle -38.66° = 19.7\angle -121.14°\text{ A}$$

$$\mathbf{I}_B = \mathbf{I}_{BR} - \mathbf{I}_{YB} = 15.63\angle -98.66° - 39.06\angle -68.66° = 26.69\angle 128.36°\text{ A}$$

These line currents are equal to the line currents of the original star-connected load.

For a star-connected load,

$$I_{ph} = I_L$$

$$\mathbf{I}_{RN} = 27.06\angle -8.65°\text{ A}$$

$$\mathbf{I}_{YN} = 19.7\angle -121.14°\text{ A}$$

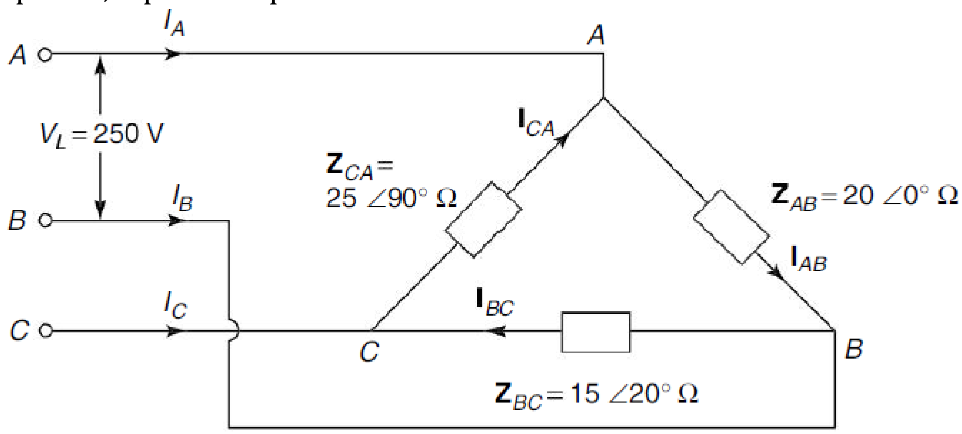
$$\mathbf{I}_{BN} = 26.69\angle 128.36°\text{ A}$$

$$\mathbf{V}_{RN} = \mathbf{Z}_R\ \mathbf{I}_{RN} = (5\angle 0°)(27.06\angle -8.65°) = 135.3\angle -8.65°\text{ V}$$

$$\mathbf{V}_{YN} = \mathbf{Z}_Y\ \mathbf{I}_{YN} = (2\angle 90°)(19.7\angle -121.14°) = 39.4\angle -31.14°\text{ V}$$

$$\mathbf{V}_{BN} = \mathbf{Z}_B\ \mathbf{I}_{BN} = (4\angle -90°)(26.69\angle 128.36°) = 106.76\angle 38.36°\text{ V}$$

13) A three-phase supply with a line voltage of 250 V has an unbalanced delta-connected load as shown in Fig. Determine (a) phase currents, (b) line currents, (c) total active power and (d) total reactive power, if phase sequence is ABC.



SOL:

Let $\mathbf{V}_{AB} = 250\angle 0^\circ$ V, $\mathbf{V}_{BC} = 250\angle -120^\circ$ V, $\mathbf{V}_{CA} = 250\angle -240^\circ$ V,

$\mathbf{Z}_{AB} = 20\angle 0^\circ\ \Omega$, $\mathbf{Z}_{BC} = 15\angle 20^\circ\ \Omega$, $\mathbf{Z}_{CA} = 25\angle 90^\circ\ \Omega$

(a) Phase currents

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{250\angle 0^\circ}{20\angle 0^\circ} = 12.5\angle 0^\circ \text{A}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = \frac{250\angle -120^\circ}{15\angle 20^\circ} = 16.67\angle -140^\circ \text{A}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = \frac{250\angle -240^\circ}{25\angle 90^\circ} = 10\angle 30^\circ \text{A}$$

(b) Line currents

$$\mathbf{I}_A = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 12.5\angle 0^\circ - 10\angle 30^\circ = 6.3\angle -52.48^\circ \text{A}$$

$$\mathbf{I}_B = \mathbf{I}_{BC} - \mathbf{I}_{AB} = 16.67\angle -140^\circ - 12.5\angle 0^\circ = 27.45\angle -157.02^\circ \text{A}$$

$$\mathbf{I}_C = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 10\angle 30^\circ - 16.67\angle -140^\circ = 26.57\angle 36.25^\circ \text{A}$$

(c) Total active power

$$P_{AB} = V_{AB}\ I_{AB}\ \cos\phi_{AB} = 250\times 12.5\times\cos(0^\circ) = 3.13 \text{ kW}$$

$$P_{BC} = V_{BC}\ I_{BC}\ \cos\phi_{BC} = 250\times 16.67\times\cos(20^\circ) = 3.92 \text{ kW}$$

$$P_{CA} = V_{CA}\ I_{CA}\ \cos\phi_{CA} = 250\times 25\times\cos(90^\circ) = 0$$

$$P = P_{AB} + P_{BC} + P_{CA} = 3.13 + 3.92 + 0 = 7.05 \text{ kW}$$

(d) Total reactive power

$$Q_{AB} = V_{AB}\ I_{AB}\ \sin\phi_{AB} = 250\times 12.5\times\sin(0^\circ) = 0$$

$$Q_{BC} = V_{BC}\ I_{BC}\ \sin\phi_{BC} = 250\times 16.67\times\sin(20^\circ) = 1.43 \text{ kVAR}$$

$$Q_{CA} = V_{CA}\ I_{CA}\ \sin\phi_{CA} = 250\times 25\times\sin(90^\circ) = 6.25 \text{ kVAR}$$

$$Q = Q_{AB} + Q_{BC} + Q_{CA} = 0 + 1.43 + 6.25 = 7.68 \text{ kVAR}$$

14) A balanced abc-sequence Y-connected source with $V_{an}=100\angle 10^0$V is connected to a Δ-connected balanced load (8+j4)Ω per phase. Calculate the phase and line currents.

SOL:

The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\underline{/26.57^\circ}\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\underline{/10^\circ}$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\underline{/30^\circ} = 100\sqrt{3}\underline{/10^\circ + 30^\circ} = \mathbf{V}_{AB}$$

$$\mathbf{V}_{AB} = 173.2\underline{/40^\circ}\text{ V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\underline{/40^\circ}}{8.944\underline{/26.57^\circ}} = 19.36\underline{/13.43^\circ}\text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 19.36\underline{/-106.57^\circ}\text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 19.36\underline{/133.43^\circ}\text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = \sqrt{3}(19.36)\underline{/13.43^\circ - 30^\circ}$$
$$= 33.53\underline{/-16.57^\circ}\text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 33.53\underline{/-136.57^\circ}\text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 33.53\underline{/103.43^\circ}\text{ A}$$

15) A 400 V, three-phase supply feeds an unbalanced three-wire, star connected load. The branch impedances of the load are Z_R = 4+j8 Ω, Z_Y = 3+j4 Ω and Z_B = 15+j20 Ω. Find the line currents and voltage across each phase impedance. Assume RYB sequence.

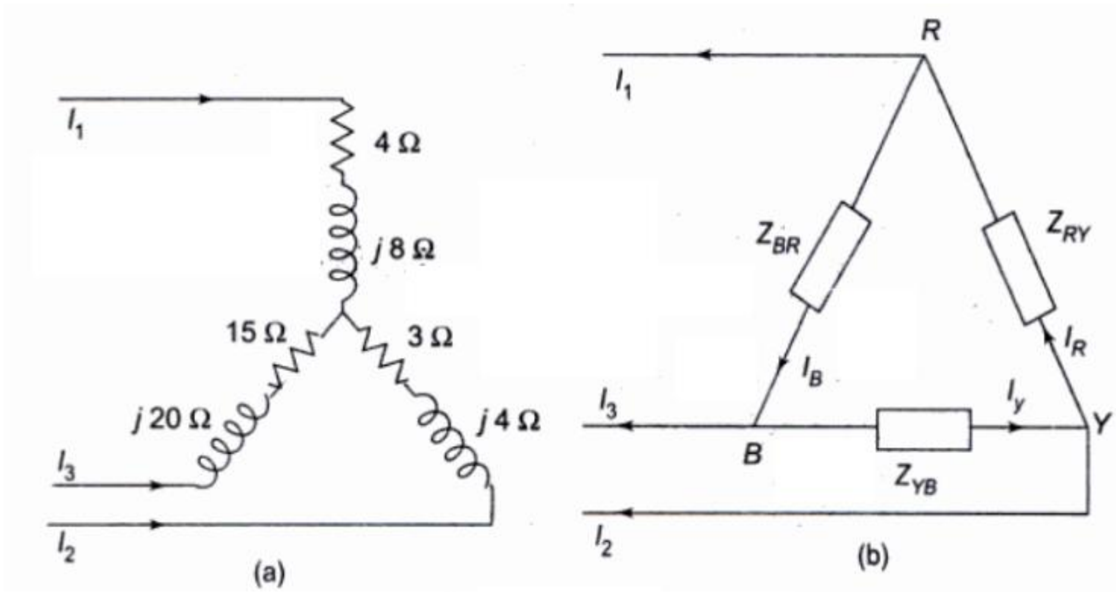
SOL:

The unbalanced star load and its equivalent delta (Δ) is shown in Fig. (a) and (b) respectively.

$$Z_R = (4 + j8)\ \Omega = 8.944\ \angle 63.4^\circ\ \Omega$$

$$Z_Y = (3 + j4)\ \Omega = 5\ \angle 53.1^\circ\ \Omega$$

$$Z_B = (15 + j20)\ \Omega = 25\ \angle 53.1^\circ\ \Omega$$



$Z_R Z_Y + Z_Y Z_B + Z_B Z_R$

$= (8.94 \angle 63.4°)(5 \angle 53.1°) + (5 \angle 53.1°)(25 \angle 53.1°)$

$+ (25 \angle 53.1°)(8.94 \angle 63.4°)$

$= 391.80 \angle 113.23°$

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} = \frac{391.80\angle 113.23°}{25\angle 53.1°} = 15.67 \angle 60.13°$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} = \frac{391.80\angle 113.23°}{8.94\angle 63.4°} = 43.83 \angle 49.83°$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} = \frac{391.80\angle 113.23°}{5\angle 53.1°} = 78.36 \angle 60.13°$$

Taking V_{RY} as reference, $V_{RY} = 400 \angle 0$

$$V_{YB} = 400 \angle -120°;\ V_{BR} = 400 \angle -240°$$

$$I_R = \frac{V_{RY}}{Z_{RY}} = \frac{400\angle 0°}{15.67\angle 60.13°} = 25.52 \angle -60.13°$$

$$I_Y = \frac{V_{YB}}{Z_{YB}} = \frac{400\angle -120°}{43.83\angle 49.83°} = 9.12 \angle -169.83°$$

$$I_B = \frac{V_{BR}}{Z_{BR}} = \frac{400\angle -240°}{78.36\angle 60.13°} = 5.10 \angle -300.13°$$

The various line currents in the delta load are

$$I_1 = I_R - I_B = 25.52 \angle -60.13° - 5.1 \angle -300.13°$$
$$= 28.41 \angle -69.07° \text{ A}$$
$$I_2 = I_Y - I_R = 9.12 \angle -169.83° - 5.52 \angle -60.13°$$
$$= 29.85 \angle 136.58° \text{ A}$$
$$I_3 = I_B - I_Y = 5.1 \angle -300.13° - 9.12 \angle -169.83°$$
$$= 13 \angle 27.60° \text{ A}$$

These line currents are also equal to the line (phase) currents of the original star-connected load. The voltage drop across each star-connected load will be as follows.

Voltage drop across $Z_R = I_1 Z_R$

$$= (28.41 \angle -69.070°)(8.94 \angle 63.4°) = 253.89 \angle -5.67° \text{ V}$$

Voltage drop across $Z_Y = I_2 Z_Y$

$$= (29.85 \angle 136.58°)(5 \angle 53.1°) = 149.2 \angle 189.68° \text{ V}$$

Voltage drop across $Z_B = I_3 Z_B$

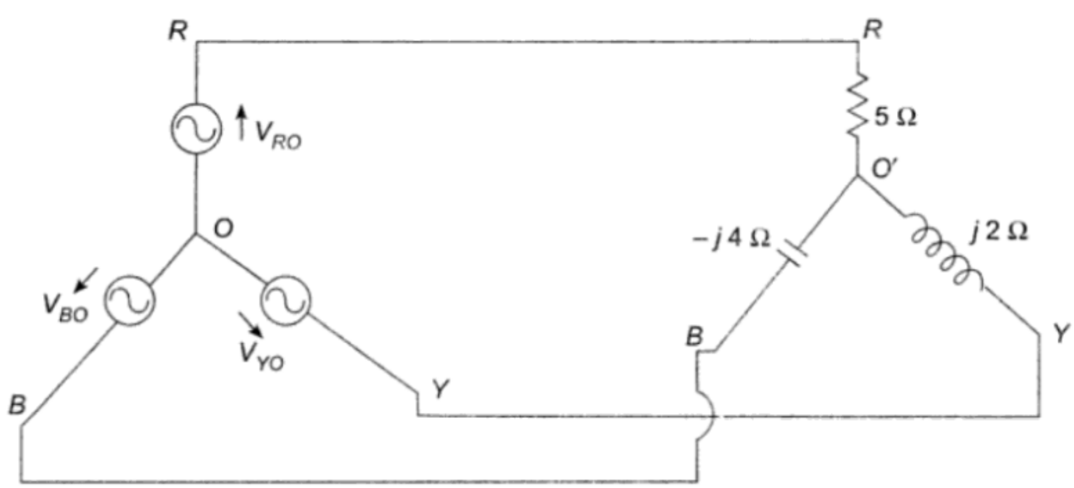
$$= (13 \angle 27.60°)(25 \angle 53.1°) = 325 \angle 80.70° \text{ V}$$

16) A symmetrical three-phase 100V; three-wire supply feeds an unbalanced star-connected load, with impedances of the load as $Z_R = 5\angle 0^0\ \Omega$, $Z_Y = 2\angle 90^0\ \Omega$ and $Z_B = 4\angle -90^0\ \Omega$. Find the (i) line currents (ii) Voltage across the impedances and (iii) the displacement neutral voltage.

SOL:

Using Millman's theorem:

Consider Fig. taking V_{RY} as reference line voltage = $100 \angle 0°$.



Phase voltages log 30° behind their respective line voltages. Therefore, the three phase voltages are

$$V_{RO} = \frac{100}{\sqrt{3}} \angle -30°$$

$$V_{YO} = \frac{100}{\sqrt{3}} \angle -150°$$

$$V_{BO} = \frac{100}{\sqrt{3}} \angle -270°$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\angle 0°} = 0.2\angle 0°$$

$$Y_Y = \frac{1}{Z_Y} = \frac{1}{2\angle 90°} = 0.5\angle -90°$$

$$Y_B = \frac{1}{Z_B} = \frac{1}{4\angle -90°} = 0.25\angle 90°$$

$$V_{RO}Y_R + V_{YO}Y_Y + Y_{BO}Y_B = (57.73 \angle -30°)(0.2 \angle 0°)$$

$$+ (57.73 \angle -150°)(0.5 \angle -90°)$$

$$+ (57.73 \angle -270°)(0.25 \angle 90°)$$

$$= 11.54 \angle -30° + 28.86 \angle -240° + 14.43 \angle -180°$$

$$= (10 - j5.77) + (-14.43 + j25) + (-14.43 + j0)$$

$$= -18.86 + j19.23 = 26.93 \angle 134.44°$$

$$Y_R + Y_Y + Y_B = 0.2 + 0.5 \angle -90° + 0.25 \angle 90°$$

$$= 0.32 \angle -51.34°$$

$$V_{O'O} = \frac{V_{RO}\,Y_R + V_{YO}\,Y_Y + V_{BO}\,Y_B}{Y_R + Y_Y + Y_B} = \frac{26.93\angle 134.44°}{0.32\angle -51.34°}$$

$$= 84.15 \angle 185.78°$$

The three load phase voltages are

$$V_{RO'} = V_{RO} - V_{O'O}$$

$$= 57.73 \angle -30° - 84.15 \angle 185.78°$$

$$= (50 - j28.86) - (-83.72 - j8.47)$$

$$= (133.72 - j20.4) = 135.26 \angle -8.67°$$

$$V_{YO'} = V_{YO} - V_{O'O}$$

$$= 57.73 \angle{-150°} - 84.15 \angle 185.78°$$

$$= (-50 - j28.86) - (-83.72 - j8.47)$$

$$= 33.72 - j20.4 = 39.4 \angle{-31.17°} \text{ or } 39.4 \angle 328.8°$$

$$V_{BO'} = V_{BO} - V_{O'O}$$

$$= 57.73 \angle{-270°} - 84.15 \angle 185.78°$$

$$= 0 + j57.73 + 83.72 + j8.47$$

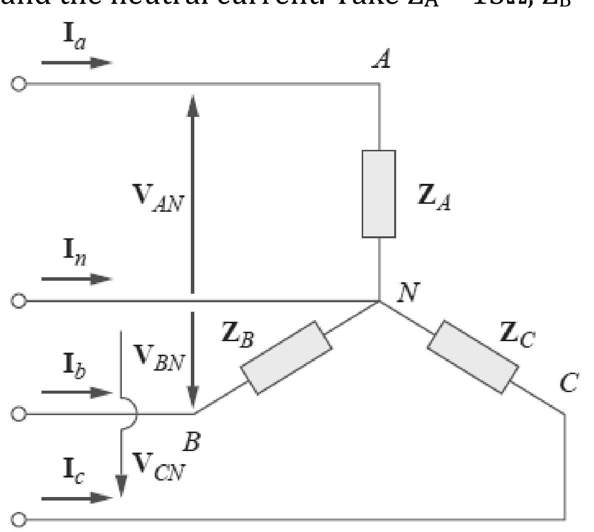
$$= 83.72 + j66.2 = 106.73 \angle 38.33°$$

$$I_R = \frac{135.26\angle{-8.67°}}{5\angle 0°} = 20.06\angle{-8.67°}$$

$$I_Y = \frac{39.4\angle 328.80°}{2\angle 90°} = 19.7\angle 238.8°$$

$$I_B = \frac{106.73\angle 38.33°}{4\angle{-90°}} = 26.68 \angle 128.33°$$

17) The unbalanced Y-load of Fig. has balanced voltages of 100 V and the acb sequence. Calculate the line currents and the neutral current. Take Z_A = 15Ω, Z_B = 10+ j5 Ω, Z_C = 6 - j8 Ω.



SOL:

the line currents are $\mathbf{I}_a = \dfrac{100\angle 0°}{15} = 6.67\angle 0° \text{ A}$

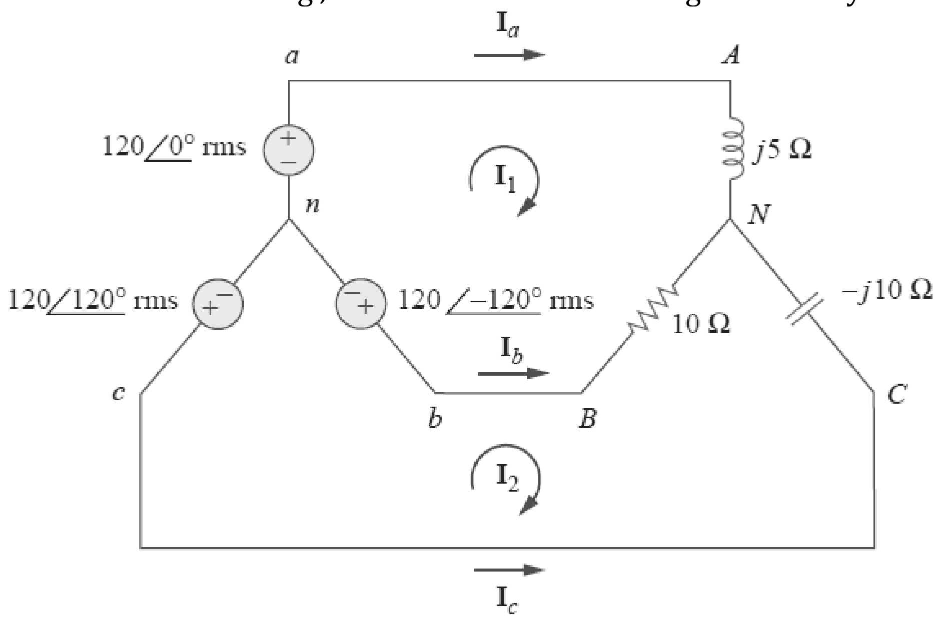
$$\mathbf{I}_b = \frac{100\angle 120^\circ}{10 + j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ} = 8.94\angle 93.44^\circ \text{ A}$$

$$\mathbf{I}_c = \frac{100\angle -120^\circ}{6 - j8} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ} = 10\angle -66.87^\circ \text{ A}$$

the current in the neutral line is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)$$
$$= -10.06 + j0.28 = 10.06\angle 178.4^\circ \text{ A}$$

18) For the unbalanced circuit in Fig., find the line currents using mesh analysis



SOL:

(a) We use mesh analysis to find the required currents. For mesh 1,

$$120\angle -120^\circ - 120\angle 0^\circ + (10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 0$$

or

$$(10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 120\sqrt{3}\angle 30^\circ$$

For mesh 2,

$$120\angle 120^\circ - 120\angle -120^\circ + (10 - j10)\mathbf{I}_2 - 10\mathbf{I}_1 = 0$$

or

$$-10\mathbf{I}_1 + (10 - j10)\mathbf{I}_2 = 120\sqrt{3}\angle -90^\circ$$

Equations form a matrix equation:

$$\begin{bmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3}\angle 30^\circ \\ 120\sqrt{3}\angle -90^\circ \end{bmatrix}$$

The determinants are

$$\Delta = \begin{vmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{vmatrix} = 50 - j50 = 70.71\angle -45^\circ$$

$$\Delta_1 = \begin{vmatrix} 120\sqrt{3}\angle 30^\circ & -10 \\ 120\sqrt{3}\angle -90^\circ & 10 - j10 \end{vmatrix} = 207.85(13.66 - j13.66)$$

$$= 4015\angle -45^\circ$$

$$\Delta_2 = \begin{vmatrix} 10 + j5 & 120\sqrt{3}\angle 30^\circ \\ -10 & 120\sqrt{3}\angle -90^\circ \end{vmatrix} = 207.85(13.66 - j5)$$

$$= 3023.4\angle -20.1^\circ$$

The mesh currents are

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{4015.23\angle -45^\circ}{70.71\angle -45^\circ} = 56.78 \text{ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{3023.4\angle -20.1^\circ}{70.71\angle -45^\circ} = 42.75\angle 24.9^\circ \text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_1 = 56.78 \text{ A}, \qquad \mathbf{I}_c = -\mathbf{I}_2 = 42.75\angle -155.1^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = 38.78 + j18 - 56.78 = 25.46\angle 135^\circ \text{ A}$$